



Microlocal and Global Analysis, Interactions with Geometry

ABSTRACTS

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MICROLOCAL AND GLOBAL ANALYSIS, INTERACTIONS WITH GEOMETRY

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<i>Chair</i>	Paycha	<i>Witt</i>	<i>Paycha</i>	<i>Schrohe</i>	<i>Ludwig</i>
9:00-9:40	Grieser	Grubb	Zenobi	Savin	Furutani
9:50-10:30	Hosseini	Tarama	Schrohe	Lesch	Kordyukov
10:40-11:00	Break				
11:00-11:40	Sjöstrand	Galkin	Ludwig	Bandara	Zanelli
11:50-12:30	Carrillo-Rouse	Schulze	Bär	Fedosova	Toft
12:30-13:20	Lunch Break				
<i>Chair</i>	<i>Bauer</i>	<i>Garmendia</i>	<i>Hosseini</i>	<i>Garmendia</i>	<i>Bandara</i>
13:20-14:00	Wei	Mazlum (book review)	Savale	Boscain	Mughetti
14:00-14:40	G (Vashakidze + Wang)	G (Molino + Sanchez)	G (Cibotaru + Kutsenko)	G (Baldare + Correa)	G (Avila Silva + Jäh)
14:50-15:30	Rowlett	Hintz	Bei	Krainer	Yuncken
15:40-16:00	Break				
16:00-16:40	Strohmaier	Popivanov	Braverman	Bulnes	Yagdjian
16:50-17:30	Behr	Mendoza	Pflaum	Rabinovich	Doll

Practical information:

- All lectures will take place online in Zoom.
- We have parallel chat rooms for questions and further discussions.
- Details will be sent by mail.

Abstracts

Alexandre Baldare

Universität Hannover

TITLE: Fredholm condition for equivariant operators.

A classical result relates the Fredholm property of a pseudodifferential operator acting on sections of a vector bundle on a closed manifold with its ellipticity. Let G be a compact Lie group and P be a G -invariant pseudodifferential operator acting between sections of G -equivariant vector bundles. With any irreducible representation α of G , we associate by restriction a G -invariant operator $\pi_\alpha(P)$ between the α -isotypical components of the Sobolev spaces. Atiyah and Singer showed that if P is G -transversally elliptic, then this operator $\pi_\alpha(P)$ is Fredholm for any irreducible representation α of G . In this talk, I will introduce the notions of α -principal symbol, transversal α -ellipticity and I will characterize Fredholmness of such operator $\pi_\alpha(P)$. This is a joint work with Rémi Come, Matthias Lesch and Victor Nistor.

Lashi Bandara

Universität Potsdam

TITLE: Fredholm and elliptic boundary conditions for elliptic differential operators on compact manifolds with boundary.

We consider boundary conditions for general order elliptic differential operators through a modern lens. Through understanding the maximal domain of the operator, we characterise Fredholm extensions as well as Elliptic extensions. The index of Fredholm operators are then related to the index of Fredholm pairs phrased in terms of the Cauchy space. We also consider spectral theory for these operators. As a special case, we consider some applications to the first-order theory.

Christian Bär

Universität Potsdam

TITLE: Spaces of metrics of positive scalar curvature on manifolds with boundary.

Unlike for closed manifolds, the existence of positive scalar curvature (psc) metrics on connected manifolds with nonempty boundary is unobstructed. We study and compare the spaces of psc metrics on such manifolds with various conditions along the boundary: $H \geq 0$, $H = 0$, $H > 0$, $\text{II} = 0$, doubling, product structure. Here H stands for the mean curvature of the boundary and II for its second fundamental form. "Doubling" means that the doubled metric on the doubled manifold (along the boundary) is smooth and "product structure" means that near the boundary the metric has product form. We show that many, but not all of the obvious inclusions are weak homotopy equivalences. In particular, we will see that if the manifold carries a psc metric with $H \geq 0$, then it also carries one which is doubling but not necessarily one which has product structure. This is joint work with Bernhard Hanke.

Malte Behr

Universität Oldenburg

TITLE: Quasihomogeneous Blow-Ups and Pseudodifferential Calculus on $SL(n, \mathbb{R})$.

We consider the *quasihomogeneous blow-up* of a submanifold Y in a surrounding manifold with corners X . It generalizes the concept of radial blow-up and revolves around the idea of assigning different *weights* to functions vanishing at the submanifold Y . This yields a filtration of the space $\mathcal{I}(Y)$ of smooth functions vanishing on Y , given by subspaces of function vanishing to a certain *quasihomogeneous order*

$$(1) \quad \mathcal{I}(Y) = \mathcal{F}^{(1)} \subseteq \mathcal{F}^{(2)} \subseteq \mathcal{F}^{(3)} \subseteq \dots$$

As an example of where such blow-ups can be used we consider the *hd-compactification* of $SL(n, \mathbb{R})$, denoted by $\overline{SL}(n, \mathbb{R})$. It was introduced by Albin, Dimakis, Melrose and Vogan for arbitrary semisimple Lie groups. $\overline{SL}(n, \mathbb{R})$ is a compact manifold with corners. On this compactification, the right-invariant vector fields may be resolved using quasihomogeneous blow-ups.

Francesco Bei

Sapienza Università di Roma

TITLE: Lp-cohomology, heat operator and stratified spaces.

Thom-Mather stratified pseudomanifolds provide an important class of singular spaces that encompasses singular complex projective varieties and orbit spaces of proper Lie groups actions. The regular part of these spaces, $\text{reg}(X)$, is generally endowed with an (iterated) conic metric g that is, an incomplete metric whose asymptotic nearby the singular locus reflects the geometry of X . Thanks to several works appeared in the last thirty years the Lp-de Rham cohomology of $(\text{reg}(X); g)$ is nowadays well known to be isomorphic (in many cases) to certain intersection cohomology groups of X . These latter groups were introduced by Goresky and MacPherson at the end of the seventies and can be thought as a sort of refinement of the usual singular homology based on the notion of “perversity” function. In this talk under suitable curvature conditions on $(\text{reg}(X); g)$ we will build an injective map (or even an isomorphism in certain cases) between the Lp and Lq de Rham cohomology groups of $(\text{reg}(X); g)$, respectively. Together with the aforementioned Lp-de Rham theorems we will show that our result can be used to produce various (in)equalities between the dimensions of intersection cohomology groups of X corresponding to different perversities.

Ugo Boscain

Sorbonne University (Paris VI)

TITLE: Quantum confinement for the curvature Laplacian on 2D-almost-Riemannian manifolds.

Two-dimension almost-Riemannian structures of step 2 are natural generalizations of the Grushin plane. They are generalized Riemannian structures for which the vectors of a local orthonormal frame can become parallel. Under the 2-step assumption the singular set Z , where the structure is not Riemannian, is a 1D embedded submanifold. While approaching the singular set, all Riemannian quantities diverge. A remarkable property of these structures is that the geodesics can cross the singular set without singularities, but the heat and the solution of the Schroedinger equation (with the Laplace-Beltrami operator Δ) cannot. This is due to the fact that (under a natural compactness hypothesis), the Laplace-Beltrami operator is essentially self-adjoint on a connected component of the manifold without the singular set. In the literature such phenomenon is called quantum confinement.

In this talk we study the self-adjointness of the curvature Laplacian, namely $-\Delta + cL$ for $c > 0$ (here K is the Gaussian curvature), which originates in coordinate free quantization procedures (as for instance in path-integral or covariant Weyl quantization). We prove that there is no quantum confinement for this type of operators.

Maxim Braverman

Northeastern University

TITLE: On Kasparov's proof of the Atiyah-Singer index theorem.

Kasparov's proof of his general KK-theoretical index theorem is quite different from most of the other approaches index theorems. In the first part of the talk, I will explain that a slight modification of Kasparov's method leads to a very elementary and straightforward proof of the classical Atiyah-Singer index theorem, which goes as follows: Given an elliptic differential operator A on a compact manifold M , we construct two globally pseudo-differential operators on $M \times \mathbb{R}^n$ whose indexes are equal to the analytical and topological index of A respectively. We then prove the Atiyah-Singer index theorem by presenting an explicit homotopy between those two operators. In the second part of the talk, I'll discuss how the aforementioned modification to the Kasparov methods simplifies many results in KK-theory. Joint work with Rudy Rodsphon.

Francisco Bulnes

IINAMEI, Tecnológico de Estudios Superiores de Chalco, México

TITLE: Micro-local Analysis on Coherent D -Modules.

We consider the Kashiwara theorem equivalences between derived categories on coherent D -modules and holomorphic G -bundles. Thanks to these, one can relate points of a moduli space and points of the Hecke eigen-sheaf of a moduli space of holomorphic vector G -bundles on a complex. We describe the moduli space constructed for characteristic cycles and their equivalences defined for functors of integral transforms by means of a space of Lagrangian submanifolds, and a complex of certain special sheaf of holomorphic G -bundles (eigen-sheaf of Hecke).

Paulo Carrillo-Rouse

Université de Toulouse III

TITLE: **On Fredholm boundary conditions on manifolds with corners and corner anomalies.**

I will report on a joint work with Jean-Marie Lescure (Paris 13). Two years ago (I won't assume you attended the talk!) I talked in this meeting on obstructions on Fredholm boundary conditions for manifolds with corners and how these obstructions lived in some very simple and computable homology groups depending on the faces. In this talk I will explicitly compute the cycles that give such obstructions showing how these cycles are constructed on what we call "corner's indices towers". These are indices of the Atiyah-Patodi-Singer type on the faces related between them by some generalized bordism relation. I will try to discuss then on how the obstruction to Fredholm boundary conditions might be interpreted as what we call corner's anomalies (notion to be explained during the talk).

Daniel Cibotaru

Universidade Federal do Ceará, Brazil

TITLE: **Bott-Chern duality and double transgressions.**

We refine the simple transgression theory of Harvey-Lawson in the complex setting. At a differential topology level this is reflected by passing from Poincaré Duality to Bott-Chern duality. One consequence of this theory is a general Poincaré-Lelong formula for a holomorphic section of a hermitian vector bundle of general rank which involves the zero locus of the section with the Hilbert-Samuel multiplicities of its irreducible components. Another application partially generalizes a result of Bismut-Gillet-Soulé from the 90's about the Chern character of a superconnection associated to a holomorphic complex chain. This is joint work with V. Grandjean and B. Lawson Jr.

David Santiago Correa Cardeno

Universität Göttingen

TITLE: **Energy estimates for weakly regular boundary problems of real type.**

Given a hyperbolic differential operator L with characteristic roots of constant multiplicities in a domain, where the boundary is non-characteristic for L , there are three generic classes of boundary conditions: 1. The uniform Lopatinskii condition holds, 2. WR (as explained in the talk), 3. The weak Lopatinskii condition fails. Case 1 is well studied. Case 3 is unstable, and there is no hope for any satisfactory theory. In this talk, we will be concerned with Case 2. We shall derive suitable energy estimates on the solutions which eventually provide the standard package of results (existence, uniqueness, higher regularity).

Fernando de Avila Silva

Federal University of Parana-Brazil

TITLE: Globally hypoelliptic systems of pseudo-differential operators on the torus T^{n+1} .

We investigate the global hypoellipticity problem for a class of systems involving pseudo-differential operators on the torus. Our approach consists in establishing conditions on the matrix symbol such that it can be transform into a suitable triangular form involving a nilpotent upper triangular matrix. We then study the global hypoellipticity by analyzing the behavior of the eigenvalues and their averages in the limit.

Moritz Doll

Universität Bremen

TITLE: Weyl Laws on Asymptotically Euclidean Manifolds.

Consider a positive elliptic SG-operator of order (m, m) with $m \in (0, \infty)$, for example $(1 + |x|^2)(1 - \Delta)$ on $\mathbb{R}^d$. Since the spectrum is purely discrete, we can consider the counting function of the eigenvalues $N(\lambda)$. The Weyl law is an asymptotic formula for $N(\lambda)$ for large λ . Together with Sandro Coriasco, we proved a 2-term Weyl law with a very precise remainder estimate improving previous results by Battisti-Coriasco and Maniccia-Panarese.

Ksenia Fedosova

Universität Freiburg

TITLE: Fourier expansion of vector-valued automorphic forms.

In this talk, we study the Fourier expansions of vector-valued eigenfunctions of the hyperbolic Laplacian that are twist-periodic. The twist may be given by any endomorphism of the finite-dimensional vector space. We are making no assumptions on invertibility or unitarity. Examples of such eigenfunctions are vector-valued twisted automorphic forms of Fuchsian groups, Eisenstein series, model resolvents etc. This is a joint work with Anke Pohl and Julie Rowlett.

Kenro Furutani

Osaka City University, Japan

TITLE: A Bargmann type transformation on the Cayley projective plane.

One method to construct the Bargmann transformation is the pairing of two polarizations on the cotangent bundle $(= T^*(\mathbb{R}^n) \cong \mathbb{C}^n)$, real and complex polarizations. In this talk, I will explain a construction of such a type transformation on Cayley projective plane, which is a symmetric space of the exceptional Lie group F_4 and is the last case remaining among projective spaces to be able to construct such a type transformation by this method. This is a report of joint work with K. Baba.

Valerii Galkin

Surgut State University, Russia

TITLE: Geometry of complicated flows in porous space containing clusters-thrombi.

One of the key problems, to which the resources of society are directed to overcome the disease caused by the new coronavirus COVID-19, is the problem of impaired blood flow, associated, in particular, with the process of thrombus formation. This phenomenon severely restricts blood flow, reducing overall oxygen delivery throughout the body. This class of problems is characterized by a complex geometry of the flow manifold, a time-varying structure of the area under consideration, a combination of problems of hydrodynamics and kinetics of coagulation of the resulting clusters-thrombi. Reconstruction of the pore space and the spread of blood clots leads to objects of set theory with properties of a fractal structure similar to the Cantorian continuum. It was noted in an article of aertzeblatt.de dating back to May 2020, that treatment with anticoagulants was associated with improved hospital survival among COVID-19 patients both in and out of the intensive care unit setting. Generally, it means that the geometry of the flow depends in great degree of space distribution of impurities, i.e. clots in porous space. The latter induce changes of flow and its interaction with porous space gives rise very complicated geometry similarly fractal sets. The work was supported by RFBR, grant 20-04-60123.

Daniel Grieser

Universität Oldenburg

TITLE: Low energy resolvent asymptotics for fibred boundary metrics.

We study the Hodge Laplacian on a non-compact complete Riemannian manifold where the metric has the structure of a fibred boundary metric at infinity. These metrics generalize metrics which are asymptotically conic (as e.g. the Euclidean metric at infinity), and arise in a number of contexts, e.g. in the theory of moduli spaces. The Hodge Laplacian is non-negative, so its resolvent is defined for negative resolvent parameter. We determine the precise asymptotic behavior of the Schwartz kernel of the resolvent as the parameter tends to zero. This is joint work with Mohammad Talebi and Boris Vertman.

Gerd Grubb

University of Copenhagen

TITLE: Recent progress in boundary value problems for fractional-order operators.

The fractional Laplacian $(-\Delta)^a$, $0 < a < 1$, is currently of great interest in probability and finance, and has applications also in differential geometry and mathematical physics. A difficulty is that it acts non-locally, in contrast to differential operators, so that problems on domains with boundary present a greater challenge than standard elliptic PDE problems. Some researchers treat it by real methods from probability and potential theory, whereas we have found the complex methods occurring in pseudodifferential operator theory more useful. The boundary value theory for this and related ps.d.o.s involves powers of the distance to the boundary.

We have recently in a joint work with Helmut Abels extended the pseudodifferential theory to allow nonsmooth boundaries. Another recent result that may be touched upon is an extension of the integration-by-parts formulas to a class of operators with both even and odd components.

Peter Hintz

MIT, Boston

TITLE: Wave decay on asymptotically flat spacetimes.

The focus of this talk is the study of asymptotic expansions of linear waves propagating on stationary and asymptotically flat spacetimes. On the spectral side, I will describe a hands-on approach to resolvent expansions near zero energy based on recent work by Vasy. I will discuss two main applications: sharp decay for scattering by short range potentials on R^3 , and a strengthening of Price's Law on Kerr spacetimes.

Mehran Seyed Hosseini

Universität Potsdam

TITLE: Moduli Space of Positive Scalar Curvature Metrics on Manifolds with Boundary and Relative L^2 -rho-invariants.

The aim of this talk is to present some results on the topology of the (moduli) space of positive scalar curvature (psc) metrics on manifolds with boundary which are collared near the boundary. After a brief introduction to the topic and statement of known results for closed manifolds, I will define a relative Cheeger-Gromov invariant associated to such metrics. I will then sketch how such invariants can be used to show that if a $4k+3$ -dimensional spin manifold with boundary admits a psc metric which is collared near the boundary and a certain algebraic condition on the fundamental groups of the manifold and its boundary is satisfied, then the moduli space of psc metrics has infinitely many path components.

Christian Jäh

Universität Rostock

TITLE: Hyperbolic systems with variable multiplicities.

We consider a Cauchy problem on $[0, T] \times R^n$ whose coefficients are matrices of t -dependent pseudodifferential operators of order 1. We suppose that the problem is hyperbolic and make no assumptions on the multiplicity of eigenvalues. After a thorough introduction, we consider the problem under an upper triangular form assumption of the coefficients. In this case, we discuss the well-posedness of the Cauchy problem in suitable Sobolev spaces as well as the propagation of singularities under suitable assumptions. The results discussed here, have been and are being worked out in collaboration with Claudia Garetto and Michael Ruzhansky.

Yuri Kordyukov

Ufa Federal Research Centre, Russia

TITLE: Bochner-Schrödinger operators on symplectic manifolds and Berezin-Toeplitz quantization.

We study a Bochner-Schrödinger operator on high tensor powers of a positive line bundle on a symplectic manifold of bounded geometry. First, we give a rough asymptotic description of its spectrum in terms of the spectra of the certain model operators. This allows us to prove the existence of gaps in the spectrum under some conditions on the Riemannian metric. Then we fix an interval whose extreme points are not in the spectrum and describe asymptotic behavior of the Schwartz kernels for the corresponding spectral projections. In the case when the manifold is compact, we develop the Toeplitz operator calculus with the quantum space, which is the eigenspace of the Bochner Laplacian with eigenvalues from the fixed interval. We show that it provides a Berezin-Toeplitz type quantization of the manifold.

Thomas Krainer

Penn State Altoona

TITLE: On Model Operators in Singular Analysis.

A common theme in PDEs is to exploit invariance properties with respect to scaling of equations. This leads to fundamental solutions, the heat kernel, as well as the notion of principal symbol. Perturbation theory is then used to derive qualitative results for more general equations, where the dominant scaling-invariant pieces are the principal parts on which invertibility assumptions (ellipticity conditions) are placed. While invertibility of the principal symbol of an elliptic operator governs certain qualitative properties of the equation locally, additional conditions are required to determine well-posedness and regularity on spaces with noncompact ends, and especially on manifolds with incomplete geometry such as boundaries and singularities (i.e. one needs to impose boundary conditions). There are operator-valued analogues of the principal symbol of the operator that are associated with the boundaries and singularities that govern the behavior of solutions and the conditions to be placed on them for the equation. These dominant terms again exhibit certain top-order homogeneity properties, i.e., scaling invariance in a suitable sense, and are sometimes referred to as model operators.

In this talk I will speak about model operators from a purely functional analytic perspective. We will obtain several results, some previously known in special cases, as well as new ones as consequences of generic functional analytic properties.

Anton Kutsenko

Jacobs Universität Bremen

TITLE: Isomorphism between one-dimensional and multidimensional finite difference operators.

Finite-difference operators are widely used for the approximation of continuous ones. It is well known that the analysis of continuous differential operators may strongly depend on their dimensions. We will show that the finite-difference operators generate the same algebra, regardless of their dimension.

Matthias Lesch

Universität Bonn

TITLE: The KO valued spectral flow for skew-adjoint Fredholm operators.

We give a comprehensive treatment of a ‘Clifford module flow’ along paths in the skew-adjoint Fredholm operators on a real Hilbert space that takes values in $KO(R)$ via the Clifford index of Atiyah–Bott–Shapiro. We develop its properties for both bounded and unbounded skew-adjoint operators including an axiomatic characterization. Our constructions and approach are motivated by the principle that spectral flow = Fredholm index. That is, we show how KO-valued spectral flow relates to a KO-valued index by proving a Robbin–Salamon type result. The Kasparov product is also used to establish a spectral flow = Fredholm index result at the level of bivariant K-theory. This is a report on joint work with Chris Bourne, Alan Carey, and Adam Rennie, completed at the University of Wollongong, Australia.

Ursula Ludwig

Universität Duisburg-Essen

TITLE: De Rham theorem for stratified spaces using Morse theory.

The famous Morse-Thom-Smale complex on a smooth compact manifold M associated to a smooth Morse function is a complex generated by the critical points of the Morse function and computes the singular (co-)homology of M . An important generalisation of this complex for smooth Morse-Bott functions is due to Austin and Braam. The Morse-Bott complex of Austin and Braam is generated by the de Rham complexes of all critical submanifolds. The aim of this talk is to adapt the construction of Austin and Braam for a stratified pseudomanifold and intersection cohomology. The main idea is to replace the de Rham complex in the construction of Austin and Braam with the complex of liftable intersection forms, due to Brasselet, Hector and Saralegi.

Gerardo Mendoza

Temple University, Philadelphia

TITLE: Polarization sets of non-degenerate almost CR structures.

The talk will begin with a general introduction to CR structures and their relevance in several complex variables. Examples of almost CR structures are complex structures and hypersurfaces in complex manifolds with the induced tangential structure. Associated to the almost CR structure there is a first order complex and the corresponding Laplacian is called Kohn Laplacian. I will show why Dencker’s notion of polarization set is relevant to study the microlocal singularities of solutions to PDEs built from such operators. I will then describe an algebra of pseudo-differential operators distilled from Beals-Greiner, Boutet de Monvel, and Sjöstrand, that contains parametrices of operators like the Kohn Laplacian, and use this to describe how the result above is obtained.

Gianmarco Vega-Molino

University of Connecticut

TITLE: **Holonomy of H-type foliations.**

We continue the study of H-type foliations initiated with F. Baudoin, E. Grong, and L. Rizzi. In this talk, we consider a notion of horizontal holonomy associated to the sub-Riemannian geometry of H-type foliations, and recover results in the direction of a Berger-Simons holonomy theorem.

Marco Mughetti

Università di Bologna

TITLE: **Analytic hypoellipticity for sums of squares operators.**

In this talk we present some recent results in the theory of real analytic regularity for the solutions to sums of squares of vector fields. While the problem of smooth hypoellipticity was settled by Hörmander, the analytic hypoellipticity remains open and seems much more involved. Trèves conjecture states that a sum of square operator is analytic hypoelliptic if and only if all the strata in the Poisson Trèves stratification are symplectic. We show that this conjecture does not hold (for the sufficient part) and we briefly discuss some model examples which would suggest that the analytic regularity of a sum of squares still depends on a suitable stratification of the characteristic variety of the operator.

Markus Pflaum

University of Colorado

TITLE: **Relative K-theory.**

For a Fréchet algebra A one can construct two natural K-theories: the topological K-theory $K^{\top}(A)$ and the algebraic K-theory $K^{alg}(A)$. In addition there exists a natural homomorphism connecting these K-theories which is called the comparison map. Karoubi invented relative K-theory as a measurement for the deviation of the comparison map to be an isomorphism which leads to a long exact sequence between the various K-theory groups. In the talk an introduction to relative K-theory for topological algebras will be given. Afterwards we report on the construction of a relative version of relative K-theory. We indicate how this relative version of relative K-theory fits into a square of K-theory groups with exact rows and columns. An application to the algebra of Whitney functions over a singular set is given and potential applications in index theory for spaces with singularities is given. The talk is based on joint work with M. Lesch and M. Martinez.

Nedyu Popivanov

Sofia University, Bulgaria

TITLE: Protter multidimensional BVP problems, singularity of the generalised solutions.

About sixty years ago Murray Protter stated a multidimensional variant of the famous Guderley-Morawetz plane problem for hyperbolic-elliptic equations that has been studied by Morawetz, Lax and Phillips. This problem now is known as Protter - Morawetz problem. Protter also gives a different statements of multidimensional analogous of the plane Darboux or Darboux-Goursat type problems for the wave equation and for some degenerated hyperbolic equations. Later appeared some analogous statements for Keldish type equations also. I will describe here the common situation in the simplest case of (3+1)D nonhomogeneous wave equation with homogeneous boundary conditions. The talk is based on the joint work with many of my coauthors, and recently with Ingo Witt, University of Göttingen.

Vladimir Rabinovich

Instituto Politécnico Nacional, ESIME Zacatenco, México

TITLE: Boundary problems for three-dimensional Dirac operators.

We consider the operators of boundary problems in unbounded domains involving a three-dimensional Dirac operators defined on the distributions as well as the magnetic and electrostatic potentials for a particle. The class of the boundary conditions we consider describe the confinement of the quarks to the domain.

We give conditions for self-adjointness of unbounded operators associated with the boundary problem under consideration and a description of its essential spectra of for some unbounded domains applying the limit operators method.

Julie Rowlett

Chalmers University of Technology, Sweden

TITLE: Same-same but different: Polyakov formulas via microlocal analysis and mathematical physics.

The zeta-regularized determinant of the Laplacian connects analysis, geometry, number theory, and physics. It is simultaneously beautiful and elusive, because in general it is impossible to compute in closed form, due to its global nature. To glean information, many have considered the variation of the determinant, because in contrast, this is a local quantity. Here, we shall investigate the variation of the determinant in singular geometric contexts. We will discuss so-called variational Polyakov formulas for surfaces and domains with singularities and boundaries. For the special cases of finite circular sectors and finite cones, we will show two different methods for obtaining the variation of the determinant. At first glance, the formulas obtained via microlocal analysis versus mathematical physics look quite different. It is an utter joy that although they appear different, we can prove that the expressions are indeed the same. This talk is based on joint work with Clara Aldana and Klaus Kirsten that was initiated a few years ago at this very same conference!

Yafet Sanchez

Universität Hannover

TITLE: Singular structure of the causal propagator.

In this talk, we will present the Sobolev wavefront set of the causal propagator for ultrastatic C^k -metrics. This is joint work with Elmar Schrohe.

Nikhil Savale

Universität Köln

TITLE: Hyperbolicity, irrationality exponents and the eta invariant.

We consider the remainder term in the semiclassical limit formula for the eta invariant on a metric contact manifold, proving in general that it is controlled by volumes of recurrence sets of the Reeb flow. This particularly gives a logarithmic improvement of the remainder for Anosov Reeb flows, while for certain elliptic flows the improvement is in terms of irrationality measures of corresponding Floquet exponents.

Anton Savin

RUDN University, Moscow

TITLE: Index Formulas for elliptic boundary value problems. Old and New.

The index problem in elliptic theory (as stated by Gelfand) requires to express indices of elliptic operators in terms of topological invariants of their principal symbols and topological invariants of manifolds on which the operators are defined. We note that index formulas in the case of pseudodifferential boundary value problems are quite different from index formulas on closed manifolds because principal symbols of the boundary value problems in question are operator functions and it is necessary to construct topological invariants which take into account these operator functions.

In this talk we explain these now classical results and show their relation to our recent results on index theory of some nonlocal elliptic boundary value problems. Our index problems originate from noncommutative geometry of Connes. In noncommutative geometry, one usually considers algebras of operators, the principal symbols of which generate essentially noncommutative algebras. Algebras of crossed product type associated with group actions on manifolds appear in many applications. The corresponding equations on the manifold include (pseudo)differential operators as well as shift operators along the orbits of group actions. For such operators, ellipticity conditions were obtained, which provide Fredholm solvability of the problem in Sobolev spaces. However, index formulas were known only in the case of smooth closed manifolds. Recently, we obtained an index formula for elliptic boundary value problems on manifolds with boundary endowed with an isometric action of a discrete group of polynomial growth in the sense of Gromov.

The results presented in this talk were obtained in a joint work with Andrei Boltachev.

The work was partially supported by RFBR grant No. 19-01-00447.

Elmar Schrohe

Universität Hannover

TITLE: The local index formula of Connes and Moscovici and equivariant zeta functions for the affine metaplectic group.

The index theory for algebras of Fourier integral operators has attracted some attention over the past decades. It encompasses a variety of interesting cases such as the Atiyah-Weinstein index problem, the index theory for operators with shifts, or the Bär-Strohmaier index theorem for Dirac operators on Lorentzian space-time manifolds and is a challenging subject on its own.

In this talk, we study a particularly simple example, namely an operator algebra on $\mathbb{R}^n$ generated by Shubin type pseudodifferential operators and affine metaplectic operators. We show how this algebra fits into the framework of local index theory developed by Connes and Moscovici. Our main result then is an explicit algebraic expression for the Connes-Moscovici cyclic cocycle. As a corollary we obtain local index formulae for noncommutative tori and toric orbifolds. (Joint work with Anton Savin, RUDN, Moscow)

Bert-Wolfgang Schulze

Universität Potsdam

TITLE: Pseudo-differential operators on singular cones.

We shall discuss families of pseudo-differential operators on an infinite straight cone CX whose link X is assumed to be singular, with piece-wise smooth geometry. More generally, we consider edges, namely locally trivial cone bundles over a smooth manifold. Such a structure is interesting in models of physics and engineering with involved corner configurations; e.g., a polyhedron in Euclidean space. The present approach is devoted to cones whose links X are stratified of depth k and CX will also be realised together with its singular conical exit to infinity.

Johannes Sjöstrand

Université de Bourgogne

TITLE: On an system of $\bar{d}$ -bar equations with a large parameter.

This talk is based on joint work with Christian Klein and Nikola Stoilov. In electrical impedance tomography and in the scattering theory of integrable Davey-Stewartson II equations, there appears a system of $\bar{d}$ -bar equations on the complex plane, which depend on a spectral parameter. We study the asymptotics of the solutions for large values of the spectral parameter. Numerical simulations show the applicability of the asymptotic formulae.

Alexander Strohmaier

University of Leeds

TITLE: **A relative trace formula in obstacle scattering.**

I will define a new relative trace for the Laplace operator with Dirichlet boundary conditions on a domain in R^n with several compact smooth obstacles on which Dirichlet boundary conditions are imposed. I will then give a trace-formula that replaces the Birman-Krein formula in this context and discuss the relation to Duistermaat-Guillemin-type trace expansions. This generalises and proves a formula used in the physics literature for the computation of Casimir forces. I also will explain the relation of our formula to Casimir force computations. (joint work with Hanisch and Waters)

Daisuke Tarama

Kyoto University, Japan

TITLE: **Integrable geodesic flow of a real semi-simple Lie group of rigid body type.**

This talk deals with a class of completely integrable geodesic flows of a real semi-simple group introduced by Mishchenko and Fomenko as a generalisation of free rigid body dynamics in analytical mechanics. The geodesic flows are defined with respect to a class of left-invariant metrics associated to Cartan subalgebras in the Lie algebra.

A brief account on the Hamiltonian formalism of the flows is given and the relative equilibria of the dynamical systems are analysed by means of the bi-Hamiltonian structure of the Hamilton equation. Then, the (Williamson) types of relative equilibria are completely classified for all the real simple Lie algebras on the basis of Kostant's classification of Cartan subalgebras in the real simple Lie algebras.

The talk is based on the joint projects with Tudor S. Ratiu at Shanghai Jiao Tong University.

Joachim Toft

Linnaeus University, Sweden

TITLE: **Analytic pseudo-differential calculus via the Bargmann transform.**

The Bargmann transform maps Fourier-invariant function spaces and their duals to certain spaces of formal power series expansions, which sometimes are convenient classes of analytic functions.

In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into an analytic pseudo-differential calculus, the so-called Wick calculus, where the involved symbols are analytic functions, and the corresponding operators map suitable classes of entire functions into other classes of entire functions. In the same manner, the Toeplitz operators correspond to so-called anti-Wick operators on the Bargmann transformed side. Recently, some investigations on certain Fourier invariant subspaces of the Schwartz space and their dual (distribution) spaces have been performed by the author. These spaces are called Pilipovic spaces, and are defined by imposing suitable boundaries on the Hermite coefficients of the involved functions or distributions. The family of Pilipovic spaces contains all Fourier invariant Gelfand-Shilov spaces as well as other spaces which are strictly smaller than any Fourier invariant non-trivial Gelfand-Shilov space. In the same way, the family of Pilipovic distribution spaces contains spaces which are strictly larger than any Fourier invariant Gelfand-Shilov distribution space.

In the talk we show that the Bargmann images of Pilipovic function and distribution spaces are convenient classes of power series expansions, and explain suitable classes of Wick operators. The talk is based on joint works with N. Teofanov and P. Wahlberg.

Zurab Vashakidze

The University of Georgia

TITLE: The non-linear dynamic Kirchhoff classical string equation.

In this talk we shall consider the classical non-linear Kirchhoff string equation. To find an approximate solution to the initial-boundary value problem for this equation we apply a three-layer symmetrical semi-discrete scheme with respect to the temporal variable, in which the value of the gradient of a non-linear term is taken at the middle point. This detail is essential because the inversion of the linear operator is sufficient for the computations of approximate solutions for each temporal steps. The variation method is applied for spatial variables and differences of the Legendre polynomials are used as coordinate functions. This way we obtain a system whose structure does not essentially differ from the corresponding system of difference equations, allowing us to use the methods developed for solving a system of difference equations.

The computer program of the numerical implementation is created based on the proposed algorithm, and numerical computations are carried out for model problems.

Changliang Wang

Tongji University, China

TITLE: Perelman's functionals on compact manifolds with conical singularities.

I will report some joint work with Prof. Xianzhe Dai about Perelman's functionals on manifolds with conical singularities. We extend some theory of the Perelman's functionals on compact smooth manifolds to compact manifolds with isolated conical singularities.

Yawei Wei

Nankai University, China

TITLE: Existence of multiple solutions for quasi-linear degenerate elliptic equations.

The present talk is concerned a class of quasi-linear elliptic degenerate equations. The degenerate operator arises from analysis of manifolds with singularities. The variational methods are applied here to verify the existence of infinity many solutions for the problem.

Karen Yagdjian

University of Texas Rio Grande Valley

TITLE: Dirac and Klein-Gordon equations in curved spacetime.

In the talk we consider the Dirac and Klein-Gordon equations in Friedmann-Lemâtre-Robertson-Walker (FLRW) spacetimes of cosmology, and discuss an integral transform approach to solve partial differential equations in curved spacetime.

Bob Yuncken

Université Clermont Auvergne

TITLE: A Rockland Theorem for sums-of-squares operators.

There are two major sources of hypoelliptic differential operators. One is Rockland operators, which are subelliptic operators on filtered Lie groups, or more generally filtered manifolds. The other is sums-of-squares operators, as in Hörmander's famous Sums-of-Squares theorem. In this talk we will introduce a pseudodifferential calculus which unifies these classes, using a singular tangent groupoid. This yields a new proof of hypoellipticity which applies to all of the above operators and many others. (Joint work with Iakovos Androulidakis, Omar Mohsen and Erik van Erp.)

Lorenzo Zanelli

University of Padova

TITLE: Mean Field asymptotics for Wick operators on Bargmann space.

We consider the flow of a discrete NLS equation arising as the Mean Field asymptotics in many body quantum theory for N interacting particles. We show Mean Field L^2 - estimates with respect to a class of invariant measures under the dNLS flow. As a consequence, we derive Hilbert-Schmidt norm estimates for quantum evolved Wick operators on the Bargmann space.

Vito Zenobi

Universität Göttingen

TITLE: Mapping the analytic surgery sequence to cyclic homology and geometric applications.

The analytic surgery sequence is a long exact sequence of K-theory groups which combines topological information (the K-homology of manifolds), index theoretic information (the K-theory of group C^* -algebras), and secondary index information (the analytic structure group of Higson-Roe).

We give a new definition of the terms, based entirely on algebras of pseudodifferential operators and their K-theory. We use this to develop systematically maps to an exact sequence of non-commutative de Rham homology/cyclic homology. Via pairings with cyclic cohomology classes, this gives rise to new numeric secondary index invariants (higher rho numbers) with explicit formulas and calculation tools due to the compatibility in the whole sequence.

We use this to for geometric applications. In particular, we derive new information about the moduli space of Riemannian metrics of positive scalar curvature, where we give new lower bounds on the number of its components.

This is joint work of Paolo Piazza, Thomas Schick, Vito Zenobi.