International Scientific Conference

Algebraic and Geometric Methods of Analysis

May 24-27, 2022, Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric and topological methods in natural sciences

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National Technological University, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

Scientific Committee

Co-Chairs:
Maksymenko S.
(*Kyiv, Ukraine*)
Prishlyak A.
(*Kyiv, Ukraine*)

Balan V.
(*Bucharest, Romania*)
Banakh T.
(*Lviv, Ukraine*)
Bolotov D.
(*Kharkiv, Ukraine*)
Cherevko Ye.
(*Odesa, Ukraine*)

Fedchenko Yu.
(*Odesa, Ukraine*)
Karlova O.
(*Chernivtsi, Ukraine*)
Kiosak V.
(*Odesa, Ukraine*)
KKonovenko N.
(*Odesa, Ukraine*)

Administrative committee

- Egorov B., chairman, rector of the ONTU;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONTU;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONTU;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies “Industry 4.0”;

Organizing commitee

Konovenko N.
Maksymenko S.
Fedchenko Yu.
Cherevko Ye.
Osadchuk E.
Sergeeva A.

On the algorithm of degenerations and fundamental groups as a tool to understand algebraic surfaces

Meirav Amram
(Shamoon College of Engineering, Ashdod, Israel)
E-mail: meiravt@sce.ac.il

The classification of algebraic surfaces in the moduli space has been an interesting question for many years. Fundamental groups are very nice invariants in classification of algebraic surfaces.

We consider an algebraic surface X in some projective space. We project X onto the projective plane $\mathbb{CP}^2$, using a generic projection, and get the branch curve S in $\mathbb{CP}^2$. The curve S is a cuspidal curve with nodes and branch points, and it can tell a lot about X. We can get the presentation of the fundamental group G of the complement of S in $\mathbb{CP}^2$. Group G does not change when the complex structure of X changes continuously. In fact, all surfaces in the same component of the moduli space have the same homotopy type and therefore have the same group G.

But it is difficult to describe S explicitly, and therefore it is not easy to write down a presentation for G. To tackle this problem, we use a nice degeneration and regeneration algorithm. And together with the use of some regeneration rules and the van-Kampen Theorem, we get the presentation of G. We note that despite these techniques, we still cannot skip some algebraic work in order to determine what G is.

If G is too complicated, we can calculate its quotient, which is the fundamental group G_{Gal} of the Galois cover of X, and also this quotient does not change when the complex structure of X changes continuously. Some examples of such calculations appear in [1] and [2]. In [1] we prove that surfaces with Zappatic singularity of type R_k have a trivial G_{Gal}. And in [2] we divide surfaces with degree 6 degenerations to two sets: trivial or non-trivial G_{Gal}. Moreover, some other works were done in this research domain, for example for surfaces with different Zappatic singularities, and surfaces that degenerate to non-planar shapes.

In the end of the talk I will present an output of a new computer algorithm, developed jointly with U. Sinichkin (TAU, Israel). This algorithm provides the presentation of the fundamental group G, when the branch curve S is given.

References

[1] Amram, M., Gong, C., Mo, J.-L.. On the Galois covers of degenerations of surfaces of minimal degree (2022).
[2] Amram, M., Gong, C., Sinichkin, U., Tan, S.-L., Xu, W.-Y., Yoshpe, M.. Fundamental groups of Galois covers of degree 6 surfaces. https://doi.org/10.1142/S1793525321500412

Topological issues about the 6D ISST in Physics

Enzo Bonacci
(The Physics Unit of ATINER, Athens, Greece)
E-mail: `enzo.bonacci@physics.org`

The recent proposals of a three-directional time [6], of a time vector [7], and of a 6D spacetime with SO(3,3) symmetry [5], have renewed the interest for the hexadimensional extension of Einstein's General Relativity formulated two decades ago via three-dimensional time [1, 2, 3]. We wish to enrich the discussion about the hypothetical 6D geometrodynamics by giving a topological response to two fundamental questions: 1) Why should the spacetime manifold require six dimensions instead of four? 2) Why should the two extradimensions be timelike? The 4D universe is supported by an intuitive logic: in order to describe an *event*, we need to know *where* and *when* it is occurring, for a total amount of four coordinates (three spatial and one temporal). Although reasonable, the current representation of the spacetime's intimate structure could be incomplete: we suggest adding the *spin angular velocity* among its intrinsic properties. If we assume that each point of the continuum is a structureless rotating sphere of null radius, we obtain a 6D inherently spinning spacetime (acronym ISST). In the ISSTconstruction, we choose to neglect both the spinning magnitude and its direction (up or down), focusing only on the plane of rotation (perpendicular to the spinning axis) as essential information about *how* an event happens. The two parameters defining the orientation of the rotation plane of a spinning point are interpreted as *time* extradimensions because they are surely not spacelike (i.e., not related to the position in a fixed *Oxyz* reference frame) and, as surface measures, they are basically timelike [4]. Our geometric analysis raises open questions ranging from the observation of a preferential arrow of time to the role of temporal "hidden variables" in classic quantum phenomena.

References

[1] Enzo Bonacci. *General Relativity Extension.* Turin : Carta e Penna, 2006.
[2] Enzo Bonacci. *Extension of Einstein's Relativity*, volume 42 of *Physical Sciences.* Rome : Aracne Editrice, 2007.
[3] Enzo Bonacci. *Beyond Relativity*, volume 43 of *Physical Sciences.* Rome : Aracne Editrice, 2008.
[4] Enzo Bonacci. Hypothetical six-dimensional reference frames. *International Journal of Mathematical Sciences and Applications*, 5(2) : 493–502, 2015.
[5] David Chester et al. Beyond the standard model with six-dimensional spacetime. *arXiv*, e-Print : 2002.02391v2 [physics.gen-ph], 2020.
[6] Günter Muchow. Is the Dimension Time Three-Directional?. *International Journal of Trend in Scientific Research and Development*, 4(5) : 514–522, 2020.
[7] Christian Rakotonirina. Energy Vector and Time Vector in the Dirac Theory. *Preprints*, 2020070472, 2020.

Characterized cycles integration on $\mathcal{D}$-modules as solutions in $\mathbb{L}$-holomorphic bundles

Francisco Bulnes
(IINAMEI A.C., Research Department in Mathematics and Engineering, TESCHA, Mexico.)
E-mail: francisco.bulnes@tesch.edu.mx

From a moduli space developed to establish the equivalences between different characteristic cycles classes; where some are objects of a complex holomorphic bundle and others elements of a sheaf of coherent $\mathcal{D}$-modules, are determined co-cycles that represent solutions of the field equations in the holomorphic context and Lagrangian submanifolds. The characteristic cycles of the category of Lagrangian submanifolds are solutions to the field equation on $\mathbb{L}$-holomorphic bundles in the space-time $\mathbb{M}$ (as complex Riemannian manifold) with singularities. We have the following technical lemma:

Lemma 1 (F. Bulnes)**.** *Characteristic cycles in $C(\mathcal{G})$, as Lagrangians have their equivalent in a flat space $\mathbb{P}^{n+4d}$, (corresponding to the spertwistor space $\mathbb{PT}$), as lines bundles in $\tilde{\mathbb{P}}$. The cycles in $C(\mathcal{G})$, are solutions of the field equation on $\mathbb{L}$-holomorphic bundles to the space-time $\mathbb{M}$, which includes singularities.*

One-dimensional Monotone Non-autonomous Dynamical Systems and Strange Nonchaotic Attractors

David Cheban
(Moldova State University)
E-mail: `david.ceban@usm.md`

This work is devoted to the study of the dynamics of one-dimensional monotone non-autonomous (cocycle) dynamical systems and strange nonchaotic attractors. A description of the structure of their invariant sets, omega limit sets, Bohr/Levitan almost periodic and almost automorphic motions, global attractors, pinched and minimal sets is given. An application of our general results is given to scalar differential and difference equations. Below we give some of our results for discrete dynamical systems generated by scalar difference equations.

Below we will use the terminology and notation from [1]. Let (Y,d) be a complete metric space and $(Y,\mathbb{Z},\sigma)$ be a dynamical system on the space Y and $C(\mathbb{Z}\times Y,\mathbb{R})$ be the space of all continuous functions $f:\mathbb{Z}\times Y\to\mathbb{R}$ equipped with the compact-open topology.

Consider the scalar difference equation

$$u(t+1)=f(\sigma(t,y),u),\quad (y\in Y) \tag{1}$$

where $f\in C(Y\times\mathbb{Z},\mathbb{R})$. Denote by $\varphi(t,u,y)$ a unique solution of equation (1) passing through the point $u\in\mathbb{R}$ at the initial moment $t=0$.

From the general properties of solutions of equation (1) we have

a. $\varphi(0,u,y)=u$ for any $u\in\mathbb{R}$ and $y\in Y$;
b. $\varphi(t+\tau,u,y)=\varphi(t,\varphi(\tau,u,y),\sigma(\tau,y))$ for any $t,\tau\in\mathbb{Z}_+$, $u\in\mathbb{R}$ and $y\in Y$;
c. the mapping $(t,u,y)\to\varphi(t,u,y)$ from $\mathbb{Z}_+\times\mathbb{R}\times Y\to\mathbb{R}$ is continuous;
d. if the function f is monotonically increasing in $u\in\mathbb{R}$ uniformly with respect to $y\in Y$, then one has $\varphi(t,u_1,y)\le\varphi(t,u_2,y)$ for any $t\in\mathbb{Z}_+$ and $y\in Y$.

Taking in consideration $a.-b.$ we can conclude that every equation (1) with monotonically increasing right hand side f generates a monotone cocycle $\langle R,\varphi,(Y,\mathbb{T},\sigma)\rangle$ with discrete time $\mathbb{Z}_+$.

Quasi-periodically forced monotone maps. An m-dimensional torus is denoted by $\mathcal{T}^m:=\mathbb{R}^m/2\pi\mathbb{Z}^m$. Let $(\mathcal{T}^m,\mathbb{T},\sigma)$ be an irrational winding of $\mathcal{T}^m$ with the frequency $\nu=(\nu_1,\nu_2,\dots,\nu_m)\in\mathbb{R}^m$. Consider difference equation

$$u(t+1)=f(\sigma(t,\omega),u), \tag{2}$$

where $f\in C(\mathcal{T}^m\times\mathbb{R},\mathbb{R})$, $\omega\in\mathcal{T}^m$ and $(\mathcal{T}^m,\mathbb{T},\sigma)$ is an irrational winding of $\mathcal{T}^m$ with the frequency $\nu=(\nu_1,\nu_2,\dots,\nu_m)\in\mathbb{R}^m$. Denote by $\varphi(t,u,\omega)$ the unique solution of equation (2) passing through the point $u\in\mathbb{R}$ ate the initial moment $t=0$. If the function f is monotonically increasing in $u\in\mathbb{R}$ uniformly with respect to $\omega\in\mathcal{T}^m$, then the mapping $\varphi:\mathbb{Z}_+\times\mathbb{R}\times\mathcal{T}^m\to\mathbb{R}$ $((t,u,\omega)\to\varphi(t,u,\omega))$ possesses the properties $a.-d.$

Theorem 1. *Let $f\in C(\mathbb{Z}\times\mathbb{R},\mathbb{R})$. Assume that the following conditions hold:*

(1) there exist a solution $\varphi(t,u_0,f)$ of equation

$$x'=f(t,x) \tag{3}$$

bounded on $\mathbb{Z}_+$;

(2) the function f is strongly Poisson stable in $t\in\mathbb{Z}$ uniformly with respect to u on every compact subset of $\mathbb{R}$.

Then the following statements hold:

(1) *the ω-limit set ω_{x_0} ($x_0 := (u_0, f) \in \mathbb{R} \times H(f)$) of point x_0 is a nonempty, conditionally compact and invariant set of skew-product dynamical system $(X, \mathbb{Z}_+, \pi)$;*
(2) *$h(\omega_{x_0}) = Y := H(f)$;*
(3) *the set ω_{x_0} contains at least one but at most two minimal sets;*
(4) *if $\mathcal{M} \subseteq \omega_{x_0}$ is a minimal set, then every point $x = (u, f) \in \mathcal{M}$ is strongly Poisson stable;*
(5) *if the function f is almost recurrent (respectively, recurrent) in $t \in \mathbb{Z}$ uniformly with respect to u on every compact subset of $\mathbb{R}$ and $\mathcal{M} \subseteq \omega_{x_0}$ is a minimal set, then every point $x = (u, f) \in \mathcal{M}$ is almost recurrent (respectively, recurrent);*
(6) *if the function f is almost automorphic in $t \in \mathbb{Z}$ uniformly with respect to u on every compact subset of $\mathbb{R}$, then the minimal set $\mathcal{M} \subseteq \omega_{x_0}$ is almost automorphic.*

Theorem 2. *Assume that equation (3) is uniformly dissipative, then the following statements hold:*
(1) *the cocycle $\langle \mathbb{R}, \varphi, (H(f), \mathbb{Z}, \sigma) \rangle$ associated by equation (3) admits a compact global attractor [2] $I = \{I_g |\ g \in H(f)\}$;*
(2) *$\alpha(g), \beta(g) \in I_g$, and hence, $I_g \subseteq [\alpha(g), \beta(g)]$, where*
$$\alpha(g) := \inf\{u \in I_g\} \quad \textit{and} \quad \beta(g) := \sup\{u \in I_g\};$$
(3) *the scalar function $\beta : H(f) \to \mathbb{R},\ g \to \beta(g)$ (respectively, $\alpha : H(f) \to \mathbb{R},\ g \to \alpha(g)$) is upper semi-continuous (respectively, lower semi-continuous);*
(4)
$$\varphi(t, \alpha(g), g) = \alpha(\sigma(t, g)) \tag{4}$$
(respectively,
$$\varphi(t, \beta(g), g) = \beta(\sigma(t, g))\ ) \tag{5}$$
for any $t \in \mathbb{Z}$ and $g \in H(f)$;
(5) *if the function f is strictly Poisson stable in $t \in \mathbb{Z}$ uniformly with respect to u on every compact subset of $\mathbb{R}$, then there exists a residual subset $G \subseteq H(f)$ such that for any $g \in G$ the solution $\varphi(t, \alpha(g), g)$ (respectively, $\varphi(t, \beta(g), g)$) of equation*
$$x' = g(t, x) \quad (g \in G \subseteq H(f)) \tag{6}$$
is compatible;
(6) *$I_g = [\alpha(g), \beta(g)]$ for any $g \in H(f)$.*

Remark 3. Suppose that $\alpha(g_0) = \beta(g_0)$ for some $g_0 \in H(f)$. Then $\alpha(g) = \beta(g)$ for a residual set $G \subseteq H(f)$ of $g \in G$. This type of attractors are called *Strange Nonchaotic Attractors* (see, for example, [3, Ch.I] and the bibliography therein).

References

[1] David N. Cheban, *Nonautonomous Dynamics:* Nonlinear Oscillations and Global Attractors. *Springer Nature Switzerland AG 2020,* xxii+ 434 pp.

[2] David Cheban, On the structure of the Levinson center for monotone dissipative non-autonomous dynamical systems. *Chapter in the book "Advance in Mathematics Research", Volume 29. Nova Science Publishers, Inc.* 2021, pp.173-218.

[3] Ulrike Feudel, Sergey Kuznetsov and Arkady Pikovsky. *STRANGE NONCHAOTIC ATTRACTORS.* Dynamics between Order and Chaos in Quasiperiodically Forced Systems. *World Scientific Series on Nonlinear Science,* Series A, Vol.56. *World Scientific,* New Jersey· London· Singapore· Beijing· Shanghai· Hong Kong· Taipei· Chennai, 2006, xi+213 pp.

Holomorphically Projective Mappings of Kähler Manifolds Preserving The Generalized Einstein Tensor

Yevhen Cherevko
(Department of Physics and Mathematics Sciences, Odesa National University of Technology 112, Kanatnaya Str., 65039, Odesa, Ukraine)
E-mail: cherevko@usa.com

Vladimir Berezovski
(Department of Mathematics and Physics, Uman National University of Horticulture 1, Institutskaya, 20300, Uman, Ukraine)
E-mail: berez.volod@gmail.com

Josef Mikeš
(Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc Křířkovského 511/8, CZ-771 47 Olomouc, Czech Republic)
E-mail: josef.mikes@upol.cz

Yuliya Fedchenko
(Department of Physics and Mathematics Sciences, Odesa National University of Technology 112, Kanatnaya Str., 65039, Odesa, Ukraine)
E-mail: fedchenko_julia@ukr.net

Holomorphically projective mappings which preserved the Einstein tensor

$$E_{ij} = R_{ij} - \frac{Rg_{ij}}{n}$$

were studied in [1]. Preserving the stress-energy tensor

$$S_{ij} = R_{ij} - \frac{Rg_{ij}}{2}$$

by conformal mappings was explored in [3], [5]. It's worth for noting that in many classical issues e. g. [2, p. 359], just the latter is referred to as the Einstein tensor.

Let us refer to

$$\mathfrak{E}_{ij} \overset{\text{def}}{=} R_{ij} - \kappa R g_{ij}. \tag{1}$$

as **the generalized Einstein tensor**. Here κ is a constant. Conformal mappings which preserving the introduced tensor were explored in [6].

It is known that a covariant vector ψ_i determining holomorphically projective mapping between two Kähler spaces (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ should satisfy the equations

$$\psi_{i,j} = \psi_i\psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{1}{n+2}(\overline{R}_{ij} - R_{ij}). \tag{2}$$

Here we denote by comma $'',''$ covariant derivative respect to the metric g of a space (V^n, J, g). The affinor J_i^h is referred to as a *complex structure*. The structure is the same for both manifolds. The symbols R_{ij} and $\overline{R}_{ij}$ denote Ricci tensors of spaces (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ respectively.

It follows from (36) that the deformation of the generalized Einstein tensor can be written as

$$\overline{\mathfrak{E}}_{ij} - \mathfrak{E}_{ij} = \overline{R}_{ij} - \kappa\overline{R}\overline{g}_{ij} - R_{ij} + \kappa R g_{ij}. \tag{3}$$

Taking account of the preservation requirement, i. e. $\overline{\mathfrak{E}}_{ij} = \mathfrak{E}_{ij}$, from (36) we get

$$\overline{R}_{ij} - R_{ij} = \kappa\overline{R}\overline{g}_{ij} - \kappa R g_{ij}. \tag{4}$$

Since (36) holds we can rewrite (36) as

$$\psi_{i,j} = \psi_i\psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{\kappa}{n+2}(\overline{R}\overline{g}_{ij} - Rg_{ij}). \tag{5}$$

Let us recall that $R = R_{ij}g^{ij}$.

Differentiating (36) covariantly with respect to x^k and the connection Γ which is compatible with the metric g of the manifold (V^n, J, g), alternating in j and k and using the Ricci identity, we obtain the conditions of integrability:

$$\psi_\alpha W^\alpha_{ijk} = \frac{\kappa}{n+2}(\partial_k\overline{R}\overline{g}_{ij} - \partial_j\overline{R}\overline{g}_{ik} - \partial_k R g_{ij} + \partial_j R g_{ik}), \tag{6}$$

where

$$W^h_{ijk} \overset{\mathrm{def}}{=} R^h_{ijk} + \frac{\kappa R}{n+2}(\delta^h_j g_{ik} - \delta^h_k g_{ij} - J^h_j J_{ik} + J^h_k J_{ij} - 2J^h_i J_{jk}). \tag{7}$$

Finally, we can summarize by the theorem

Theorem 1. *If manifolds (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ are in holomorphically projective correspondence and the mapping preserves the tensor $\mathfrak{E}_{ij} = R_{ij} - \kappa R g_{ij}$, then the function ψ generating the mapping, must satisfy the system of PDE's* (36) *whose conditions of integrability are* (36)*. Also, the tensor W^h_{ijk} is preserved by the mapping.*

References

[1] Chepurna O., Mikeš J. Holomorphically projective mappings preserving the Einstein tensor. *J. Appl. Math. Aplimat* 2011, 4:2, 187-192.

[2] Petrov A. Z. New methods in the general theory of relativity, *Nauka* 1966, 496 p.

[3] Conformal mappings preserving the stress-energy tensor *Izvestia PSPU* 2011, vol. 26, p. 98-104.

[4] Chepurna O., Kiosak V., Mikeš J. Conformal mappings of Riemannian spaces which preserve the Einstein tensor *J. Appl. Math. Aplimat* 2010, 3:1, p. 253-258.

[5] Cherevko Y. V. Conformal transformations preserving the stress-energy tensor *Proc. Inter. Geom. Center* 2012, vol. 5, no. 3-4, p. 51-58.

[6] Cherevko Y., Chepurna O., Berezovski V., Conformal mappings of Riemannian manifolds preserving the generalized Einstein tensor *APLIMAT 2018 - Proceedings* 2018, p. 224-231.

Some questions about virtual Legendrian knots

Vladimir Chernov
(Dartmouth College, Hanover, USA)
E-mail: vladimir.chernov@dartmouth.edu

Rustam Sadykov
(Kansas State University)
E-mail: rstsdk@gmail.com

Virtual Legendrian knots were introduced by Cahn and Levi and jointly with Sadykov we proved the Kuperberg type theorem for them. We will discuss a few open questions about the virtual Legendrian knots including the versions of the Ding-Geiges Theorem, Arnolds 4 cusp conjecture and the applications of this to causality in spacetimes with the changing topology of the spacelike section in the spirit of our works with Nemirovski.

Morse-Smale flows in the Boy's surface

Luca Di Beo
(Taras Shevchenko National University of Kyiv, Kyiv, Ukraine)
E-mail: dibeoluca@gmail.com

Morse-Smale (MS) dynamical systems are amongst the simplest possible dynamical systems, with strong restrictions imposed on its critical points. In this thesis, I present a brief history of the development of the theory, along with the introduction of important definitions, theorems and lemmas. Moreover, I investigate MS systems in the Boy's surface (P‘) with emphasis on optimal ones. A method relying only on topological features has been used in order to classify MS systems in P‘. A review of some topological properties of this space is presented in order to construct the necessary arguments that allowed the discovery of this type of flow in P‘.

At the time this thesis was written, there was no current work in the literature regarding the classification of all optimal MS flows in P‘. Hence, my original contribution to knowledge here is the finding of all 342 optimal MS flows in P‘, the finding of all 80 optimal Projective MS (PMS) flows (Projective MS flows in P‘are those MS flows in P‘that can be extended to MS flow in $\mathbf{R}P^2$) in P‘, and the exposure of a few non-optimal ones, as a preparatory path for future researchers, all up to symmetry.

Morita equivalence of non-commutative Noetherian schemes

Yuriy Drozd
(Institute of Mathematics NASU)
E-mail: y.a.drozd@gmail.com

This is a joint work with Igor Burban, see [1].

The classical Morita theorem (see, for instance, [3, Ch. 18]) claims that the categories of modules over rings A and B are equivalent if and only if there is a finitely generated projective generator P of the category of right A-modules such that $\mathrm{End}_A P \simeq B$. Then this equivalence is established by the functor $P \otimes_A -$. If A and B are Noetherian, the same is the criterion of equivalence of their categories of finitely generated modules. On the other hand, Gabriel [2] proved that two Noetherian schemes X and Y are isomorphic if and only if the categories of coherent (or, which is the same, of quasi-coherent) sheaves of $\mathcal{O}_X$- and $\mathcal{O}_Y$-modules are equivalent. We present here a result which is, in some sense, a combination and generalization of these two classical theorems.

Definition 1. (1) A *non-commutative Noetherian scheme* (*NCNS*) is a pair $\mathbb{X} = (X, \mathcal{O}_\mathbb{X})$, where X is a separated Noetherian scheme and $\mathcal{O}_\mathbb{X}$ is a sheaf of $\mathcal{O}_X$-algebras which is coherent as a sheaf of $\mathcal{O}_X$-modules. We denote by $\mathrm{Coh}\,\mathbb{X}$ and $\mathrm{QCoh}\,\mathbb{X}$ respectively the categories of coherent and quasi-coherent sheaves of left $\mathcal{O}_\mathbb{X}$-modules.

Note that the category $\mathrm{QCoh}\,\mathbb{X}$ is locally Noetherian and $\mathrm{Coh}\,\mathbb{X}$ is its subcategory of Noetherian objects. Therefore, they uniquely define each other.

(2) Two NCNS $\mathbb{X}$ and $\mathbb{Y}$ are called *Morita equivalent* if the categories $\mathrm{Coh}\,\mathbb{X}$ and $\mathrm{Coh}\,\mathbb{Y}$ (or, which is the same, $\mathrm{QCoh}\,\mathbb{X}$ and $\mathrm{QCoh}\,\mathbb{Y}$) are equivalent.

(3) A NCNS $\mathbb{X}$ is called *central* if $\mathcal{O}_X$ coincides with the center of $\mathcal{O}_\mathbb{X}$, i.e. for every point $x \in X$ the ring $\mathcal{O}_{X,x}$ is the center of the algebra $\mathcal{O}_{\mathbb{X},x}$.

Proposition 2. *For every NCNS $\mathbb{X} = (X, \mathcal{O}_\mathbb{X})$ there is a Noetherian scheme Z and a morphism $\phi : Z \to X$ such that the NCNS $\tilde{\mathbb{X}} = (Z, \phi^*\mathcal{O}_\mathbb{X})$ is central and Morita equivalent to $\mathbb{X}$. Moreover, the ring of global sections $\Gamma(Z, \mathcal{O}_Z)$ is isomorphic to the* center *of the category* $\mathrm{Coh}\,\mathbb{X}$*, i.e. the endomorphism ring of the identity functor* $\mathrm{id}_{\mathrm{Coh}\,\mathbb{X}}$*. If the scheme X is excellent, the morphism ϕ is finite.*

Thus, studying Morita equivalence, we can only consider central schemes. The following result is an analogue of the Gabriel's theorem.

Theorem 3. *If a NCNS $\mathbb{X} = (X, \mathcal{O}_\mathbb{X})$ is central, the scheme X is determined by the category* $\mathrm{QCoh}\,\mathbb{X}$ *(or, which is the same, by* $\mathrm{Coh}\,\mathbb{X}$*) up to an isomorphism.*

Actually, we give an explicit construction that restores X from $\mathrm{QCoh}\,\mathbb{X}$, namely, from the so called *spectrum* of this category in the sense of Gabriel [2], i.e. isomorphism classes of indecomposable injective objects. It is important that this construction also recovers affine open coverings of X.

Definition 4. A coherent sheaf of right $\mathcal{O}_\mathbb{X}$-modules $\mathcal{P}$ is called a *local progenerator* for $\mathbb{X}$ if for every point $x \in X$ its stalk $\mathcal{P}_x$ is a projective generator of the category of right $\mathcal{O}_{\mathbb{X},x}$-modules.

Our main result if the following.

Theorem 5. *Let $\mathbb{X} = (X, \mathcal{O}_\mathbb{X})$ and $\mathbb{Y} = (Y, \mathcal{O}_\mathbb{Y})$ be central NCNS. They are Morita equivalent if and only if there is an isomorphism $\phi : Y \to X$ and a local progenerator $\mathcal{P}$ for $\mathbb{X}$ such that $\phi^*(\mathcal{E}nd_{\mathcal{O}_\mathbb{X}} \mathcal{P}) \simeq \mathcal{O}_\mathbb{Y}$. Then this equivalence is established by the functor $\phi^*(\mathcal{P} \otimes_{\mathcal{O}_\mathbb{X}} -)$.*

Note that even if $X = Y$, the isomorphism ϕ need not be identity. If it is so, this equivalence is called *central*.

We also specialize this theorem for the case of *non-commutative curves*, where it gives a sort of "globalization" of the known results on the local–global correspondence from the theory of lattices over orders (or integral representations of rings).

Definition 6. A *non-commutative curve* is a NCNS $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ such that X is excellent and of pure dimension 1 and $\mathcal{O}_{\mathbb{X}}$ is *reduced*, i.e. contains no nilpotent ideals.

We always suppose $\mathbb{X}$ *central* and *connected* (in the central case, it just means that X is connected). We denote by $\mathcal{Q}_X$ the sheaf of fractions of $\mathcal{O}_X$ and set $\mathcal{Q}_{\mathbb{X}} = \mathcal{Q}_X \otimes_{\mathcal{O}_X} \mathcal{O}_{\mathbb{X}}$. We denote $Q(X) = \Gamma(X, \mathcal{Q}_X)$ and $Q(\mathbb{X}) = \Gamma(X, \mathcal{Q}_{\mathbb{X}})$. Note that $Q(\mathbb{X})$ is a semisimple $Q(X)$-algebra and for every closed point $x \in X$ the ring $\mathcal{O}_{\mathbb{X},x}$ is an $\mathcal{O}_{X,x}$-order in this algebra. Since X is excellent, the set $\mathrm{Sing}(\mathbb{X})$ of such closed points $x \in X$ that this order is not maximal is finite (it follows from [4, Ch. 6]).

Theorem 7. *Let $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ and $\mathbb{Y} = (X, \mathcal{O}_{\mathbb{Y}}))$ be two central non-commutative curves with the same central curve X. They are centrally Morita equivalent if and only if the following conditions are satisfied:*

- *the semisimple $Q(X)$-algebras $Q(\mathbb{X})$ and $Q(\mathbb{Y})$ are centrally Morita equivalent;*
- $\mathrm{Sing}(\mathbb{X}) = \mathrm{Sing}(\mathbb{Y})$*;*
- *for every $x \in \mathrm{Sing}(\mathbb{X})$ the $\mathcal{O}_{X,x}$-orders $\mathcal{O}_{\mathbb{X},x}$ and $\mathcal{O}_{\mathbb{Y},x}$ (or, which is the same, their $\mathfrak{m}_x$-completions) are centrally Morita equivalent.*

References

[1] I. Burban, Yu. Drozd. Morita theory for non-commutative noetherian schemes. Adv. Math., ?? : ???–???, 2022.

[2] P. Gabriel. Des catégories abéliennes. Bull. Soc. Math. France, 90 : 323–448, 1962. https://doi.org/10.24033/bsmf.1583.

[3] T. Lam. *Lectures on Modules and Rings*, Springer–Verlag, New York, 1999.

[4] I. Reiner. *Maximal Orders*, The Clarendon Press, Oxford University Press, Oxford, 2003.

Some critical point results for Fréchet manifolds

Kaveh Eftekharinasab
(Institute of Mathematics of NAS of Ukraine)
E-mail: kaveh@imath.kiev.ua

Linking techniques (see [1]) provide significant results in critical points theory. We present linking theorem and some of its corollaries, namely a mountain pass theorem and a three critical points theorem for Keller C^1-functional on C^1-Fréchet manifolds. We refer to [2] for the definitions.

Theorem 1 (Linking Theorem, [2]). *Let M be a C^1- Fréchet manifold endowed with a complete Finsler metric ρ and let $\varphi : M \to \mathbb{R}$ be a closed Keller C^1_c-functional. Suppose $\{S_0, S, C\}$ is a linking set through $\gamma \in C(S_0, \mathsf{T})$, C is closed and $\rho(\gamma(S_0), C) > 0$. Suppose the following conditions hold*

(1) $\mathbf{s} :- \sup_{\gamma(S_0)} \leq \inf_C \varphi -: \mathbf{i}$,

(2) φ satisfies the Palais-Smale condition at

$$c :- \inf_{h \in \mathcal{H}} \sup_{x \in S} \varphi(\gamma(x)), \tag{1}$$

where $\mathcal{H} :- \{h \in C(S, \mathsf{T}) : h|_{\partial S_0} = \gamma\}$.

Then c is a critical value and $c \geq \mathbf{i}$. Furthermore, if $c = \mathbf{i}$ then $\mathrm{Cr}(\varphi, c) \cap C \neq \emptyset$.

The theorem yields the following corollaries:

Theorem 2 (Mountain Pass Theorem, [2]). *Suppose that $x_0, x_1 \in M$, x_0 belongs to an open subset $U \subset M$ and $x_1 \notin \overline{U}$. Let $\varphi : M \to \mathbb{R}$ be a closed a Keller C^1_c-functional satisfying the following condition:*

(1) $\max\{\varphi(x_0), \varphi(x_1)\} \leq \inf_{\partial U} \varphi(x) :- \mathbf{i}$*;*

(2) φ satisfies the Palais-Smale condition at

$$c :- \inf_{h \in \mathcal{C}} \sup_{t \in [0,1]} \varphi(h(t)), \tag{2}$$

where $\mathcal{C} :- \{h \in C([0,1], M) : h(0) = x_0,\ h(1) = x_1\}$.

Then c is a critical value and $c \geq \mathbf{i}$. If $c = \mathbf{i}$ then $\mathrm{Cr}(\varphi, c) \cap U \neq \emptyset$.

Theorem 3 (Three Critical Points Theorem, [2]). *Let M be a connected Fréchet manifold and $\varphi : M \to \mathbb{R}$ a closed a Keller C^1_c-functional satisfying the Palais-Smale condition at all levels. If φ has two minima, then φ has one more critical point.*

We apply the mountain pass theorem and the Minimax principle to prove the following theorem which provides the sufficient conditions for a local diffeomorphism to be a global one.

Theorem 4. [2] *Let M, N be connected C^1- Fréchet manifolds endowed with complete Finsler metrics δ, ρ respectively. Assume that $\varphi : M \to N$ is a local diffeomorphism of class Keller C^1_c. Let $\mathcal{I} : N \to [0, \infty]$ be a closed Keller C^1_c-functional such that $\mathcal{I}(x) = 0$ if and only if $x = 0$ and $\mathcal{I}'(x) = 0$ if and only if $x = 0$. If for any $q \in N$ the functional ϕ_q defined by*

$$\phi_q(x) = \mathcal{I}(\varphi(x) - q)$$

satisfies the Palais-Smale condition at all levels, then φ is a Keller C^1_c-global diffeomorphism.

References

[1] Martin Schetcher. Linking Method in Critical Point Theory. Berlin Birkhäuser Basel, 1999.
[2] Kaveh Eftekharinasab. *Some critical point results for Fréchet manifolds. https://arxiv.org/abs/2205.01359.*

On partial preliminary group classification of some class of $(1+3)$-dimensional Monge-Ampere equations. One-dimensional Galilean Lie algebras.

Vasyl Fedorchuk
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine, 79060, 3-b Naukova St., Lviv, Ukraine)
E-mail: vasfed@gmail.com

Volodymyr Fedorchuk
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine, 79060, 3-b Naukova St., Lviv, Ukraine)
E-mail: volfed@gmail.com

A solution of many problems of the geometry, theoretical physics, astrophysics, differential equations, nonlinear elasticity, fluid dynamics, optimal mass transportation, one-dimensional gas dynamics and etc. has reduced to investigation of classes of Monge-Ampère equations in the spaces of different dimensions and different types. At the present time, there are a lot of papers and books in which those classes have been studied by different methods.

Let us consider the following class of $(1+3)$-dimensional Monge-Ampére equations:

$$\det\left(u_{\mu\nu}\right) = F\left(x_0, x_1, x_2, x_3, u, u_0, u_1, u_2, u_3\right),$$

where $u = u(x)$, $x = (x_0, x_1, x_2, x_3) \in M(1,3)$, $u_{\mu\nu} \equiv \dfrac{\partial^2 u}{\partial x_\mu \partial x_\nu}$, $u_\alpha \equiv \dfrac{\partial u}{\partial x_\alpha}$, $\mu, \nu, \alpha = 0, 1, 2, 3.$

Here, $M(1,3)$ is a four-dimensional Minkowski space, F is an arbitrary real smooth function.

For the group classification of this class we have used the classical Lie-Ovsiannikov approach. At the present time, we have performed partial preliminary group classification of the class under consideration, using one-dimensional nonconjugate Galilean subalgebras of the Lie algebra of the Poincaré group $P(1,4)$.

In my report, I plan to present some of the results obtained concerning with partial preliminary group classification of the class under consideration.

References

[1] G. Ţiţeica. Sur une nouvelle classe de surfaces. *Comptes Rendus Mathématique. Académie des Sciences. Paris*, 144 : 1257–1259, 1907.

[2] A.V. Pogorelov. *The multidimensional Minkowski problem.* Moscow : Nauka, 1975.

[3] Shing-Tung Yau, Steve Nadis. *The shape of inner space. String theory and the geometry of the universe's hidden dimensions.* New York : Basic Books, 2010.

[4] D.V. Alekseevsky, R. Alonso-Blanco, G. Manno, F. Pugliese. Contact geometry of multidimensional Monge-Ampère equations:characteristics, intermediate integrals and solutions. *Ann. Inst. Fourier (Grenoble)*, 62(2) : 497–524, 2012.

[5] C. Enache. Maximum and minimum principles for a class of Monge-Ampère equations in the plane, with applications to surfaces of constant Gauss curvature. *Commun. Pure Appl. Anal.*, 13(3) : 1347–1359, 2014.

[6] Haiyu Feng, Shujun Shi. Curvature estimates for the level sets of solutions to a class of Monge-Ampère equations. *Nonlinear Anal.*, 178 : 337–347, 2019.

[7] S.V. Khabirov. Application of contact transformations of the inhomogeneous Monge-Ampère equation in one-dimensional gas dynamics. *Dokl. Akad. Nauk SSSR*, 310(2) : 333–336, 1990.

[8] D.J. Arrigo, J.M. Hill. On a class of linearizable Monge-Ampère equations. *J. Nonlinear Math. Phys.*, 5(2) : 115–119, 1998.

[9] F. Jiang, Neil S. Trudinger. On Pogorelov estimates in optimal transportation and geometric optics. *Bull. Math. Sci.*, 4(3) : 407–431, 2014.

[10] A. Figalli. *The Monge-Ampère equation and its applications. Zurich Lectures in Advanced Mathematics.* Zurich : European Mathematical Society (EMS), 2017.
[11] S. Lie. On integration of a class of linear partial differential equations by means of definite integrals. *Arch. Math*, 6(3) : 328–368, 1881.
[12] L.V. Ovsiannikov. Group properties of the equation of non-linear heat conductivity. *Dokl. Akad. Nauk SSSR* , 125 : 492–495, 1959.
[13] L.V. Ovsiannikov. *Group analysis of differential equations.* Moscow : Nauka, 1978.
[14] V.V. Lychagin, V.N. Rubtsov, I.V. Chekalov. A classification of Monge-Ampère equations. *Ann. Sci. École Norm. Sup. (4)*, 26(3) : 281–308, 1993.
[15] D. Tseluiko. On classification of hyperbolic Monge-Ampère equations on 2-dimensional manifolds. *Rend. Sem. Mat. Messina Ser. II*, 8(23) : 139–150, 2004.
[16] A. De Paris, A.M. Vinogradov. Scalar differential invariants of symplectic Monge-Ampère equations. *Cent. Eur. J. Math.*, 9(4) : 731–751, 2011.
[17] V.I. Fushchich, N.I. Serov. Symmetry and some exact solutions of the multidimensional Monge-Ampère equation. *Dokl. Akad. Nauk SSSR*, 273(3) : 543–546, 1983.
[18] V.I. Fushchich, V.M. Shtelen, N.I. Serov. *Symmetry analysis and exact solutions of nonlinear equations of mathematical physics.* Kiev : Naukova Dumka, 1989.
[19] C. Udrişte, N. Bîlă. Symmetry group of Ţiţeica surfaces PDE. *Balkan J. Geom. Appl.*, 4 : 123–140, 1999.
[20] V.M. Fedorchuk, V.I. Fedorchuk. On classification of the low-dimensional nonconjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$. *Proc. of the Inst. of Math. of NAS of Ukraine.* Kyiv : Institut of Mathematics of NAS of Ukraine, 3(2) : 302–308, 2006.

On packing and lattice packing of Minkowski-Chebyshev balls

Nikolaj Glazunov
(Glushkov Institute of Cybernetics NASU, Kiev)
E-mail: glanm@yahoo.com

The Minkowski hypothesis was formulated in [1] and refined in [2, 3, 4]. Regarding the concepts of the geometry of numbers, see [5].

Let

$$D_p = \{(x,y),\ p > 1\} \subset \mathbb{R}^2 \tag{1}$$

be the 2-dimension region:

$$|x|^p + |y|^p < 1. \tag{2}$$

Let

$$\Delta(p,\sigma) = (\tau + \sigma)(1+\tau^p)^{-\frac{1}{p}}(1+\sigma^p)^{-\frac{1}{p}}, \tag{3}$$

be the function defined in the domain

$$\mathcal{M}:\ \infty > p > 1,\ 1 \le \sigma \le \sigma_p = (2^p - 1)^{\frac{1}{p}}, \tag{4}$$

of the $\{p,\sigma\}$ plane, where σ is some real parameter; here $\tau = \tau(p,\sigma)$ is the function uniquely determined by the conditions

$$A^p + B^p = 1,\ 0 \le \tau \le \tau_p,$$

where

$$A = A(p,\sigma) = (1+\tau^p)^{-\frac{1}{p}} - (1+\sigma^p)^{-\frac{1}{p}}, \tag{5}$$

$$B = B(p,\sigma) = \sigma(1+\sigma^p)^{-\frac{1}{p}}\tau(1+\tau^p)^{-\frac{1}{p}}, \tag{6}$$

τ_p is defined by the equation

$$2(1-\tau_p)^p = 1 + \tau_p^p,\ 0 \le \tau_p \le 1. \tag{7}$$

Proposition 1. *The function $\Delta(p,\sigma)$ in region $\mathcal{M}$ determines the moduli space of admissiblel lattices of the rigion D_p each of which contains three pairs of points on the boundary of D_p.*

Proposition 2. *Let $\Delta(D_p)$ be the critical determinant of the region $|x|^p + |y|^p < 1$. Let $\Lambda_p^{(0)}$ and $\Lambda_p^{(1)}$ be two D_p-admissible lattices each of which contains three pairs of points on the boundary of D_p and with the property that $(1,0) \in \Lambda_p^{(0)}$, $(-2^{-1/p}, 2^{-1/p}) \in \Lambda_p^{(1)}$. Under these conditions the lattices are uniquely defined.*

Let $d(\Lambda_p^{(0)}), d(\Lambda_p^{(1)})$ be determinants of the lattices. Let $\Delta_p^{(1)} = \Delta(p,1) = 4^{-\frac{1}{p}}\frac{1+\tau_p}{1-\tau_p}$, $\Delta_p^{(0)} = \Delta(p,\sigma_p) = \frac{1}{2}\sigma_p$.

Proposition 3. $d(\Lambda_p^{(0)}) = \Delta(p,\sigma_p),\ d(\Lambda_p^{(1)}) = \Delta(p,1).$

Remark 4. For example in the case $p = 2$ the lattice $\Lambda_2^{(0)}$ has the determinant $d(\Lambda_2^{(0)}) = \frac{\sqrt{3}}{2}$ and is defined by generators $a_1 = (1,0), a_2 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Theorem 5. [6]

$$\Delta(D_p) = \begin{cases} \Delta(p,1),\ 1 < p \leq 2,\ p \geq p_0, \\ \Delta(p,\sigma_p),\ 2 \leq p \leq p_0; \end{cases}$$

here p_0 is a real number that is defined unique by conditions $\Delta(p_0, \sigma_p) = \Delta(p_0, 1),\ 2,57 \leq p_0 \leq 2,58$.

Definition 6. In two-dimensional case we will call geometric figures of the form $|x|^p + |y|^p \leq R, R > 0$, with $p \geq p_0$ the two-dimensional Minkowski-Chebyshev balls.
In cases of dimension grater then two, when the constant p_0 is unknown, we will call geometric figures of the form $|x_1|^p + |x_2|^p + |x_3|^p + \cdots + |x_n|^p \leq R, R > 0$, the n-dimensional Minkowski-Chebyshev balls if p is a sufficiently large.

We investigate packing and lattice packing by equal Minkowski-Chebyshev balls of n-dimensional Euclidean spaces and also of corresponding spheres.

Proposition 7. *Let $\mathbb{Z}^2$ be the integer lattice in $\mathbb{R}^2$ with a point in the origin. Then the density of packing by two-dimensional open Minkowski-Chebyshev balls over the lattice $\mathbb{Z}^2$ tends to unity as p tends to infinity*

Conjecture 8. *Let Λ be the integer $(n > 2)$-dimensional lattice in $\mathbb{R}^n$ with a point in the origin. Then the density of packing by n-dimensional open Minkowski-Chebyshev balls over the lattice Λ tends to unity as p tends to infinity*

Problem 9. Is there an analogue of Theorem 5 in the case of geometric bodies of the form

$$|x_1|^p + |x_2|^p + |x_3|^p + \cdots + |x_n|^p < 1, n > 2,$$

.

Problem 10. If there exists an analogue of Theorem 5 in the case of geometric bodies of the form

$$|x_1|^p + |x_2|^p + |x_3|^p + \cdots + |x_n|^p < 1, n > 2,$$

what is the value of the constant p_0 .

References

[1] H. Minkowski, *Diophantische Approximationen*, Leipzig: Teubner (1907).
[2] C. Davis, Note on a conjecture by Minkowski, *J. London Math. Soc.*, **23**, 172–175 (1948).
[3] H. Cohn, Minkowski's conjectures on critical lattices in the metric $\{|\xi|^p + |\eta|^p\}^{1/p}$, *Annals of Math.*, **51**, (2), 734–738 (1950).
[4] G. Watson, Minkowski's conjecture on the critical lattices of the region $|x|^p + |y|^p \leq 1$, (I), (II), *Jour. London Math. Soc.*, **28**, (3, 4), 305–309, 402–410 (1953).
[5] J. Cassels, *An Introduction to the Geometry of Numbers*, Berlin: Springer-Verlag (1997).
[6] N. Glazunov, A. Golovanov, A. Malyshev,
Proof of Minkowski's hypothesis about the critical determinant of $|x|^p + |y|^p < 1$ domain , Research in Number Theory 9. Notes of scientific seminars of LOMI. **151**(1986), Nauka, Leningrad, 40–53.

Unbounded order and norm convergence of some operators on Banach lattices

Omer GOK
(Yildiz Technical University, Faculty of Arts and Sciences, Mathematics Department, Esenler, Istanbul, TURKEY)
E-mail: gok@yildiz.edu.tr

Let X be a Banach space. An operator $T : X \to X$ is said to be demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n). For example, each compact operator is demicompact. But, the converse is not true in general. If the identity operator $I : X \to X$ on the infinite dimensional Banach space X, then $-I$ is demicompact but it is not compact. We say that an operator $T : X \to X$ is weakly demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ weakly converges in X, there is a weakly convergent subsequence of (x_n). Every demicompact operator is weakly demicompact. An operator $T : X \to Y$ between Banach spaces is called Dunford-Pettis if it carries weakly compact subsets of X onto compact subsets of Y. Equivalently, for each weakly null sequence (x_n) we have $\|Tx_n\| \to 0$ as $n \to \infty$. An operator $T : X \to X$ is called unbounded demi Dunford-Pettis if, for every sequence (x_n) in X such that $x_n \to 0$ in $\sigma(X, X')$ and $(x_n - Tx_n)$ unbounded norm converges to 0 as $n \to \infty$, we have (x_n) unbounded norm convergent to 0. For example, for the identity operator $I : l^\infty \to l^\infty$, $-I$ is unbounded demi Dunford-Pettis operator.

Theorem 1. *Let E be a Banach lattice. Every Dunford-Pettis operator $T : E \to E$ is unbounded demi Dunford-Pettis.*

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demicompactness property.

References

[1] C.D. Aliprantis, O. Burkinshaw. *Positive Operators*Academic Press, London, 1985.
[2] H.Benkhaled, A. Elleuch, A. Jeribi, The class of order weakly demicompact operators, *RACSAM*, **114**, 76, 2020.
[3] Y.A. Dabboorasad, E.Y. Emelyanov, M.A.A. Marabeh, $u\tau$-convergence in locally solid vector lattices, *Positivity*, **22**, 1065-1080, 2018.
[4] W.V. Petryshyn, Construction of fixed points of demicompact mappings in Hilbert space, *J. Math. Anal. Appl.*,**14**, 276-284,1966.

An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$

Ji-Young Ham
(Department of Architecture, Konkuk University, 120 Neungdong-ro, Gwangjin-gu, Seoul, Korea 05029)
E-mail: jiyoungham1@gmail.com
Joongul Lee
(Department of Mathematics Education, Hongik University, 94 Wausan-ro, Mapo-gu, Seoul, Korea 05029)
E-mail: jglee@hongik.ac.kr

An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$ up to repeated factors is presented.

The main purpose of the paper is to find the explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$ up to repeated factors. Let us denote the knot with Conway's notation $C(2n,4)$ by T_{2n} and the A-polynomial of the knot with Conway's notation $C(2n,4)$ by A_{2n}. The following theorem gives the explicit formula for the A-polynomial of T_{2n}.

Theorem 1. *The A-polynomial $A_{2n}=A_{2n}(L,M)$ is given explicitly by*

$$A_{2n}=p_{2n}(u)p_{2n}(-u)$$

where

$$p_{2n}(z)=\begin{cases}\sum_{i=0}^{2n}\binom{\lfloor\frac{i}{2}\rfloor+n}{i}2^{-2\lfloor\frac{i+1}{2}\rfloor-i}\left(M^2\right)^{-\lfloor\frac{i}{2}\rfloor-2\lfloor\frac{i+1}{2}\rfloor+i+n}\left(LM^2+1\right)^{-2\lfloor\frac{i+1}{2}\rfloor-i+2n}\\ \times\left(-2LM^6+LM^4-LM^2-M^4+M^2z+M^2-2\right)^{\lfloor\frac{i+1}{2}\rfloor}\\ \times\left(LM^2+L+M^2+z+1\right)^i\left(-3LM^2+L+M^2+z-3\right)^{\lfloor\frac{i-1}{2}\rfloor}\\ \times\left((-1)^{i+1}\left(LM^2+1\right)-2LM^2+L+M^2+z-2\right) & \text{if } n\geq 0,\\ \sum_{i=0}^{-2n}\binom{\lfloor\frac{i-1}{2}\rfloor-n}{i}2^{-2\lfloor\frac{i+1}{2}\rfloor-i}\left(M^2\right)^{-\lfloor\frac{i}{2}\rfloor-2\lfloor\frac{i+1}{2}\rfloor+i-n}\left(LM^2+1\right)^{-\frac{1}{2}-2\lfloor\frac{i+1}{2}\rfloor-i-2n}\\ \times\left(-2LM^6+LM^4-LM^2-M^4+M^2z+M^2-2\right)^{\lfloor\frac{i+1}{2}\rfloor}\\ \times\left(LM^2+L+M^2+z+1\right)^i\left(-3LM^2+L+M^2+z-3\right)^{\lfloor\frac{i-1}{2}\rfloor}\\ \times\left((-1)^i\left(-2LM^2+L+M^2+z-2\right)-LM^2-1\right) & \text{if } n<0,\end{cases}$$

and

$$u=\sqrt{5L^2M^4-2L^2M^2+L^2-2LM^4+12LM^2-2L+M^4-2M^2+5}.$$

Acknowledgement: This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. NRF-2018R1A2B6005847). The second author was supported by 2018 Hongik University Research Fund.

References

[1] D. Cooper, M. Culler, H. Gillet, D. D. Long, and P. B. Shalen. Plane curves associated to character varieties of 3-manifolds. *Invent. Math.*, 118(1):47–84, 1994.
[2] Daniel V. Mathews. An explicit formula for the A-polynomial of twist knots. *J. Knot Theory Ramifications*, 23(9), 2014.

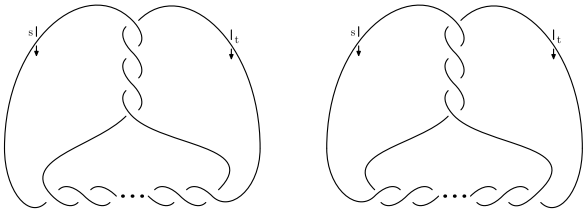

FIGURE 1.1. A two bridge knot with Conway's notation $C(2n, 4)$ for $n > 0$ (left) and for $n < 0$ (right)

[3] Ji-Young Ham and Joongul Lee. An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n, 3)$. *J. Knot Theory Ramifications*, 25(10):1650057, 9, 2016.

[4] Ji-Young Ham and Joongul Lee. Explicit formulae for Chern-Simons invariants of the twist-knot orbifolds and edge polynomials of twist knots. *Mat. Sb.*, 207(9):144–160, 2016.

[5] Ji-Young Ham, Joongul Lee, Alexander Mednykh, and Aleksei Rasskazov. On the volume and Chern-Simons invariant for 2-bridge knot orbifolds. *J. Knot Theory Ramifications*, 26(12):1750082, 22, 2017.

The symplectic invariants of 3-webs

Konovenko Nadiia
(ONTU, Odesa, Ukraine)
E-mail: ngkonovenko@gmail.com

The classical web geometry ([1],[2],[4]) studies invariants of foliation families with respect to pseudogroup of diffeomorphisms. Thus for the case of planar 3-webs the basic semi invariant is the Blaschke curvature ([3]). It is also curvature of the Chern connection ([4]) that are naturally associated with a planar 3-web.

Let $\mathbf{D} \subset \mathbb{R}^2$ be a connected and simply connected domain in the plane, equipped with symplectic structure given by differential 2-form $\Omega = dx \wedge dy$ in the standard coordinates on the plane.

Remind that a 3-web in the domain is a family of three foliations being in general position. We'll assume that these foliations are integral curves of differential 1-forms ω_i, $i = 1, 2, 3$, and write

$$W_3 = \langle \omega_1, \omega_2, \omega_3 \rangle,$$

where $\omega_i \in \Omega^1(\mathbf{D})$ are such differential 1-forms that $\omega_i \wedge \omega_j \neq 0$ in $\mathbf{D}$, if $i \neq j$.

Definition 1. We say that two planar 3-webs W_3 and $\widetilde{W_3}$ given in domains $\mathbf{D}$ and $\widetilde{\mathbf{D}}$ respectively are symplectively equivalent if there is a symplectomorphism $\phi : \mathbf{D} \to \widetilde{\mathbf{D}}$, such that $\phi(W_3) = \widetilde{W_3}$.

Proposition 2. *Let $W_3 = \langle \omega_1, \omega_2, \omega_3 \rangle$ and $\widetilde{W_3} = \langle \widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\omega}_3 \rangle$ be two planar 3-webs in domains $\mathbf{D}$ and $\widetilde{\mathbf{D}}$ respectively given by normalized*

$$\omega_1 + \omega_2 + \omega_3 = 0. \tag{1}$$

differential forms. Then a diffeomorphism $\phi : \mathbf{D} \to \widetilde{\mathbf{D}}$ establishes a symplectic equivalence of 3-webs if and only if

$$\phi^*(\widetilde{\omega}_i) = \varepsilon \omega_{\sigma(i)},$$

where $(\sigma, \varepsilon) \in \mathbb{A}_3 \times \mathbb{Z}_2$, and $\mathbb{A}_3 \subset \mathbb{S}_3$ is the subgroup of even permutations and $\mathbb{Z}_2 = \{1, -1\}$.

In our case normalization (1) and the above proposition shows that the Chern form γ is itself symplectic invariant of 3-webs.

Let's write down γ in following form

$$\gamma = x_1\omega_1 + x_2\omega_2 + x_3\omega_3,$$

where functions $x_i \in C^\infty(\mathbf{D})$ are barycentric coordinates of γ, i.e.

$$x_1 + x_2 + x_3 = 1.$$

Then we have

$$\begin{aligned} d\omega_1 &= (x_3 - x_2)\, \omega_1 \wedge \omega_2, \\ d\omega_2 &= (x_1 - x_3)\, \omega_1 \wedge \omega_2, \\ d\omega_3 &= (x_2 - x_1)\, \omega_1 \wedge \omega_2. \end{aligned}$$

Using the second normalization (1) condition we'll rewrite these relations in the following form

$$\begin{aligned} d\omega_i &= \lambda_i \Omega, \quad i = 1, 2, 3, \\ \lambda_1 &= x_3 - x_2, \lambda_2 = x_1 - x_3, \lambda_3 = x_2 - x_1, \end{aligned} \tag{2}$$

and

$$x_1 = \frac{1}{3}(1 + \lambda_2 - \lambda_3), x_2 = \frac{1}{3}(1 + \lambda_3 - \lambda_1), x_3 = \frac{1}{3}(1 + \lambda_1 - \lambda_2).$$

Theorem 3. *Functions*

$$\begin{aligned} J_1 &= \lambda_1^2+\lambda_2^2+\lambda_3^2, \\ J_2 &= \lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2, \\ J_w &= \left(\lambda_2^2-\lambda_1^2\right)\left(\lambda_3^2-\lambda_1^2\right)\left(\lambda_3^2-\lambda_2^2\right) \\ J_3 &= \lambda_1^2\lambda_2^2\lambda_3^2 \end{aligned}$$

are symplectic invariants of 3-webs.

References

[1] Akivis M. A., "Differential geometry of webs", Itogi Nauki i Tekhniki. Ser. Probl. Geom., 15, VINITI, Moscow, 1983, 187–213; J. Soviet Math., 29:5

[2] Blaschke W., Bol G.. Geometrie der Gewebe. Vol. 335. No. 8. Berlin, 1938.

[3] Blaschke W. Einführung in die Geometrie der Waben, Birkhäuser 1955

[4] Chern S.-S., Web geometry. Bulletin of the American Mathematical Society 6.1 (1982) : 1-8.

Geometric interpretation of first Betti numbers of orbits of smooth functions

Iryna Kuznietsova
(Department of Algebra and Topology, Institute of Mathematics of NAS of Ukraine,
Tereshchenkivska str. 3, Kyiv, 01024, Ukraine)
E-mail: kuznietsova@imath.kiev.ua

Yuliia Soroka
(Department of Algebra and Topology, Institute of Mathematics of NAS of Ukraine,
Tereshchenkivska str. 3, Kyiv, 01024, Ukraine)
E-mail: sorokayulya@imath.kiev.ua

Let M be a compact connected surface and P is a real line $\mathbb{R}$ or a circle S^1. Denote by $\mathcal{F}(M,P)$ the space of smooth functions $f \in C^{\infty}(M,P)$ satisfying the following conditions:

1) the function f takes constant value at ∂M and has no critical point in ∂M;
2) for every critical point z of f there is a local presentation $f_z \colon \mathbb{R}^2 \to \mathbb{R}$ of f near z such that f_z is a homogeneous polynomial $\mathbb{R}^2 \to \mathbb{R}$ without multiple factors.

Let X be a closed subset of M. Denote by $\mathcal{D}(M,X)$ the group of C^{∞}-diffeomorphisms of M fixed on X, that acts on the space of smooth functions $C^{\infty}(M,P)$ by the rule: $(f,h) \longmapsto f \circ h$, where $h \in D(M,X)$, $f \in C^{\infty}(M,P)$.

The subset $\mathcal{S}(f,X) = \{h \in D(M,X) \mid f \circ \ h = f\}$ is called the *stabilizer* of f with respect to the action above and $\mathcal{O}(f,X) = \{f \circ h \mid h \in D(M,X)$ is *orbit* of f. Denote by $\mathcal{D}_{id}(M,X)$ the identity path component of $\mathcal{D}(M,X)$ and let $\mathcal{S}'(f,X) = \mathcal{S}(f) \cap \mathcal{D}_{id}(M,X)$.

Homotopy types of stabilizers and orbits of Morse functions were calculated in a series of papers by Sergiy Maksymenko, Bohdan Feshchenko, Elena Kudryavtseva and others. Furthermore, precise algebraic structure of such groups for the case $M \neq S^2, T^2$ was described in [1]. In particular, the following theorem holds.

Theorem 1. [1] *Let M be a connected compact oriented surface except 2-sphere and 2-torus and let $f \in \mathcal{F}(M,P)$. Then $\pi_0\mathcal{S}'(f,\partial M) \in \mathcal{B}$, where $\mathcal{B}$ is a minimal class of groups satisfying the following conditions:*

1) $1 \in \mathcal{B}$;
2) if $A, B \in \mathcal{B}$, then $A \times B \in \mathcal{B}$;
3 if $A \in \mathcal{B}$ and $n \geq 1$, then $A \wr_n \mathbb{Z} \in \mathcal{B}$.

Note that a group G belongs to the class $\mathcal{B}$ iff G is obtained from trivial group by a finite number of operations $\times$, $\wr_n\mathbb{Z}$. It is easy to see that every group $G \in \mathcal{B}$ can be written as a word in the alphabet $\mathcal{A} = \{1, \mathbb{Z}, (,), \times, \wr_2, \wr_3, \wr_4, \dots\}$. We will call such word a *realization* of the group G in the alphabet $\mathcal{A}$.

Denote by $\beta_1(G)$ the number of symbols $\mathbb{Z}$ in the realization ω of group G. The number $\beta_1(G)$ is the rank of the center $Z(G)$ and the quotient-group $G/[G,G]$ (Theorem 1.8 [2]). Note, this number depends only on the group G, not the presentation ω. Moreover, $\beta_1(G)$ is first Betti number of $\mathcal{O}(f)$.

Edge of Γ_f will be called *external* if it is incident to the vertex of Γ_f that is corresponding to a non-degenerate critical point of f or non-fixed boundary component of ∂M with respect to the action of $S'(f,W)$ for f-adapted submanifold X which contains $W = S^1 \times 0$. Otherwise, it will be called *internal*. Denote by $\sharp Orb_{int}(M,W)$ the number of orbits of the action of $S'(f,W)$ on internal edges of $\Gamma_{f|_X}$.

Theorem 2. *Let M be a disk D^2 or a cylinder $C = S^1 \times [0,1]$ and $f \in \mathcal{F}(M,P)$. Then*

$$\sharp Orb_{int}(M,W) = \beta_1(\pi_0 S^{'}(f,\partial M)),$$

where $W = \partial M$ if $M = D^2$ or $W = S^1 \times 0$ if M is a cylinder.

References

[1] Maksymenko S. I. Deformations of functions on surfaces by isotopic to the identity diffeomorphisms. *Topology and its Applications*, vol. 282, 2020.

[2] Kuznietsova, I.V. and Soroka, Y.Y. The first Betti numbers of orbits of Morse functions on surfaces. *Ukrainian Math. Journal*, 73(2): 179–200, 2021.

On diffeological principal bundles of non-formal pseudo- differential operators over formal ones

Jean-Pierre Magnot
(Univ. Angers, CNRS, LAREMA, SFR MATHSTIC, F-49000 Angers, France and Lycée Jeanne d'Arc, Avenue de Grande Bretagne, 63000 Clermont-Ferrand, France)
E-mail: `magnot@math.cnrs.fr`

Let E be a complex vector space over a compact boundaryless manifold M. In this communication, G denotes either the group of non-formal, invertible bounded classical pseudodifferential operators or the group of invertible elements of the algebra of non-formal, maybe unbounded, classical pseudodifferential operators of integer order, equipped with a given diffeology which makes classical composition and inversion smooth. H is the normal subgroup of G of operators which are equal to Id up to a smoothing operator. We also assume that the group H is regular for its subgroup diffeology. We analyze the short exact sequence

$$Id \to H \to G \to G/H \to Id,$$

where G/H is understood as a group of formal pseudodifferential operators, along the lines of the theory of principal bundles, where, G is the total space, G/H is the base space and H is the structure group.

Problem 1. There is actually no local slice $G/H \to G$, or in other words the principal bundle $G \to G/H$ has no known local trivialization.

Therefore, one has to consider what has been called by Souriau as "structure quantique" in [4] and diffeological connections along the lines of Iglesias-Zemmour [1] in order to interpret the so-called smoothing connections described in [2] (that we generalize here for S^1 to any M) in terms of horizontal paths. More precisely, we show:

Theorem 2. *Any smoothing connection in the sense of* [2] *defines a diffeological connection along the lines of* [1].

and we explain how one can understand the notion of curvature of covariant derivatives, with values in smoothing operators, in terms of curvature of a connection 1-form on $G \to G/H$.

Then, we specialize to $M = S^1$, by giving more examples of smoothing connections, and explain in this context how the Schwinger cocyle is, in cohomology, a first Chern form of the principal bundle $G \to G/H$ for a given smoothing connection. We finish the exposition of the results by showing that higher Chern forms $tr(\Omega^k)$ of this connection with curvature Ω define closed $2k-$cocycles on the Lie algebra of G, and that the cocycle obtained for $k = 2$ is non trivial, along the lines of [3].

As a conclusion, we give open problems related both to our construction and to the interpretation of index-like problems on pseudodfferential operators.

References

[1] Iglesias-Zemmour, P.; *Diffeology* Mathematical Surveys and Monographs **185** AMS, 2013.

[2] Magnot, J-P.; On the geometry of $Diff(S^1)-$pseudodifferential operators based on renormalized traces. *Proceedings of the International Geometry Center* 14 (1) : 19-47 (2021) . https://doi.org/10.15673/tmgc.v14i1.1784

[3] Magnot, J-P.; On a class of closed cocycles for algebras of non-formal, possibly unbounded, pseudodifferential operators `ArXiv:2012.06941`

[4] Souriau, J.M.; Un algorithme générateur de structures quantiques; *Astérisque*, Hors Série, 341-399, 1985.

Topological actions of wreath products

Sergiy Maksymenko
(Algebra and Topology department, Institute of mathematics, NAS of Ukraine, Str. Tereshchenkivska, 3, Kyiv, 01024, Ukraine)
E-mail: `maks@imath.kiev.ua`

Let G and H be two groups acting on path connected topological spaces X and Y respectively. Assume that H is finite of order m and the quotient maps $p : X \to X/G$ and $q : Y \to Y/H$ are regular coverings. Then it is well-known that the wreath product $G \wr H$ naturally acts on $W = X^m \times Y$, so that the quotient map $r : W \to W/(G \wr H)$ is also a regular covering. We give an explicit description of $\pi_1(W/(G \wr H))$ as a certain wreath product $\pi_1(X/G) \wr_{\partial_Y} \pi_1(Y/H)$ corresponding to a *non-effective* action of $\pi_1(Y/H)$ on the set of maps $H \to \pi_1(X/G)$ via the boundary homomorphism $\partial_Y : \pi_1(Y/H) \to H$ of the covering map q.

Such a statement is known and usually exploited only when X and Y are contractible, in which case W is also contractible, and thus $W/(G \wr H)$ is the classifying space of $G \wr H$.

The applications are given to the computation of the homotopy types of orbits of typical smooth functions f on orientable compact surfaces M with respect to the natural right action of the groups $\mathcal{D}(M)$ of diffeomorphisms of M on $\mathcal{C}^\infty(M, \mathbb{R})$.

References

[1] S. Maksymenko, *Topological actions of wreath products*, arXiv:1409.4319v3, 2022, 24 pages

The geometrical properties of degenerations of curves and surfaces

Jia-Li Mo
(School of Mathematics, Soochow University, China.P.R)
E-mail: mojiali0722@126.com

In this talk, we will mainly discuss the topology and arithmetic properties of degenerations of curves and surfaces. First, we investigate the influences of the base points of cubic pencils on the Mordell-Weil groups in this part. We pay attention to 8, 7 ,6 and 5 base points in general position for such cubic pencil, and classify these cubic pencils. And we give the following theorem:

Theorem 1. *This is the main theorem (taken from* [2]*).*
Given $n\ (=8,7,6,5)$ *points in general position in* $\mathbb{P}^2$, $S: sH_1+tH_2=0,\ \ [s,t]\in\mathbb{P}^1$ *is a cubic pencil with* $n\ (=8,7,6,5)$ *simple base points. Then, the Mordell-Weil groups of the fibrations are isomorphic to two types respectively:*

$$E_8:\quad y^2=x^3+x(\sum_{i=0}^{3}p_it^i)+\sum_{i=0}^{3}q_it^i+t^5,\quad y^2=x^3+t^2x^2+x(\sum_{i=0}^{2}p_it^i)+\sum_{i=0}^{4}q_it^i+t^5 \tag{1}$$

$$E_7^\vee:\quad y^2=x^3+x(p_0+p_1t+t^3)+\sum_{i=0}^{4}q_it^i,\quad y^2+txy=x^3+x(\sum_{i=0}^{2}p_it^i)+\sum_{i=0}^{3}q_it^i-t^4 \tag{2}$$

$$E_6^\vee:\quad y^2+t^2y=x^3+x(\sum_{i=0}^{2}p_it^i)+(\sum_{i=0}^{2}q_it^i),\quad y^2+txy=x^3+x(\sum_{i=0}^{2}p_it^i)+(\sum_{i=0}^{3}q_it^i) \tag{3}$$

$$D_5^\vee:\quad y^2+p_5xy=x^3+p_4tx^2+(p_8t^2+p_2t^3)x+p_6t^4+t^5 \tag{4}$$

A Del Pezzo surface X is either $\mathbb{P}^1\times\mathbb{P}^1$ or the blow-up of $\mathbb{P}^2$ in $m\ (m=1,\cdots,8)$ points in general position. The degree d of X is defined to be $d=9-m$. As an application, we give a new proof of the number of (-1) curves in Del Pezzo surfaces.

Theorem 2. *The number of* (-1) *curves in Del Pezzo surfaces of degree* $1,2,3,4$ *is 240, 56, 27 and 16 respectively.*

In the second part, we talk about the surfaces of minimal degree in $\mathbb{P}^n$. In fact, the degree of such surface is $n-1$. The fundamental group of Galois cover of surface is an important invariant of the moduli space of such surfaces. In [1], we use the tools of degenerations of surfaces to prove the following theorem:

Theorem 3. *The Galois cover of the surface of minimal degree is simple-connected and general type.*

In the end, we give an open question:
Question: It is well known that the fundamental groups of most surfaces of general type are non commutative. But it is not easy to find concrete examples of such surfaces. Let a_k be a series of integral number whose limit is infinity. How to give a series of surfaces of degree a_k whose the fundamental groups of Galois covers are all non commutative?

References

[1] Amram, M., Gong, C., Mo, J.-L., *On the Galois covers of degenerations of surfaces of minimal degree*, appear in Mathematische Nachrichten, 2021.
[2] Mo, J.-L., *The Mordell-Weil groups of cubic pencils*, reprint.

Nilpotent aaproximations in the Goursat Monster Tower

Piotr Mormul
(University of Warsaw, Warsaw, POLAND)
E-mail: `mormul@mimuw.edu.pl`

In the paper *"Kumpera–Ruiz algebras in Goursat flags are optimal in small lengths"* (J. Math. Sciences **126** (2005), 1614 − 1629) we conjectured that the two notions 'strongly nilpotent' (Definition 3 up there) and 'tangential' (Definition 6 up there) are but synonyms in the world of Goursat flags. Now a concrete road map to a possible proof of that long-standing conjecture is being proposed.

The role of topological invariants in the study of the early evolution of the Universe

Tetiana Obikhod
(03028, Institute for Nuclear Research, Prospect Nauki, 47, Kiev, Ukraine)
E-mail: obikhod@kinr.kiev.ua

Questions of the evolution of the Universe, the nature of forces and physical processes at an early stage of the evolution of the Universe are the most relevant in theoretical high-energy physics. The evolution of the Universe is connected with phase transitions in vacuum, represented by alternating minima and maxima of the potential. The discovery of the Higgs boson led to the problem of a metastable vacuum in the mechanism of electroweak symmetry breaking and confirmed the hypothesis that a vacuum decay took place. Such a transition in vacuum between two minima can be represented in D-brane language. D-brane approach is realized through Planck brane in the left minimum of potential and TeV brane in the right minimum of potential. Every D-brane presented in terms of vector bundle is characterized by topological invariant, [1]. So, the calculation of topological invariants informs us about the possibility of phase transitions between different states of vacuum.

We considered two universal bundles $\alpha_2^5 : (V_2(R^5), p, G_2(R^5)), \alpha_2^6 : (V_2(R^6), p, G_2(R^6))$ which are isomorphic to vector bundles, γ_2^5, γ_2^6 correspondingly. Taking into account the theorem on the existence of a vector bundle $V_{\rho(n)+1}(R^n) \to S^{n-1}$, [2] for $n > 4$, and using the fact

$$\frac{PR^{n-1}}{PR^{n-2}} \to V_1(R^n) = S^{n-1}$$

we presented the exact sequence

$$0 \to \pi_3(V_1(R^4)) \to \pi_4(V_1(R^5)) = Z.$$

We used the equivalence of homotopic groups

$$\pi_3(V_1(R^4)) = \pi_3(V_2(R^5))$$
$$\pi_4(V_1(R^5)) = \pi_4(V_2(R^6))$$

according to [2] with $F = R, c = 1, k = 1$.

Using the fact that D-branes can be represented as a vector bundles with a base - a sphere and using the embedding of spheres, $S^4 \subset S^5$, we observe a transition from one solitonic state in the form of D5-brane to D4-brane with the corresponding equidistant set of energy levels. The obtained result signals about the possibility of phase transitions in the form of vacuum decay from Planck brane to TeV brane.

References

[1] Paul S. Aspinwall. *D-Branes on Calabi-Yau Manifolds*, arXiv:hep-th/0403166.
[2] D.Husemoller. *Fibre bundles*, *McGraw-Hill Book Co., New York-London-Sidney*, 1966.

O-spheroids in metric and linear normed spaces

Illia Ovtsynov
(Taras Shevchenko National University of Kyiv, Kyiv, Ukraine)
E-mail: iliarkov@gmail.com

Definition 1. Open O-spheroid with rank n, or O-spheroid with rank n, in a metric space (X, ρ) with a metric ρ, $n \in \mathbb{N}$, is a set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) < a\},$$

where $x_1, \ldots, x_n$ are different fixed points of the space (X, ρ), called the foci, and a is a fixed positive number, called the distance. We can get a respective definition in linear normed spaces.

Definition 2. Closed O-spheroid with rank n in a metric space (X, ρ) with a metric ρ, $n \in \mathbb{N}$, is a set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) \leq a\},$$

where $x_1, \ldots, x_n$ are different fixed points of the space (X, ρ), called the foci, and a is a fixed positive number, called the distance. We can get a respective definition in linear normed spaces.

Remark 3. $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is an open O-spheroid with rank n with the foci in points $x_1, \ldots, x_n$ and the distance a. If we talk about open O-spheroid understanding what namely O-spheroid we discuss, we note it $\mathbb{S}_n$.

Definition 4 ([11, c. 193])**.** Border of (open or closed) O-spheroid with rank n, or n-ellipse with the foci $x_1, \ldots, x_n$ and the distance a, in a metric space (X, ρ) we name the set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) = a\}.$$

Definition 5. Focal closeness of our O-spheroid with rank n equals to

$$\pi(\mathbb{S}_n(x_1, \ldots, x_n; a)) := \min_{1 \leq i < j \leq n} \rho(x_i, x_j).$$

Definition 6. Focal remoteness of our O-spheroid with rank n equals to

$$\Phi(\mathbb{S}_n(x_1, \ldots, x_n; a)) := \max_{1 \leq i < j \leq n} \rho(x_i, x_j).$$

Definition 7. If all the foci belong to O-spheroid, then it is called a *multicentered* one.

Theorem 8. *Let's assume we have an O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ in a metric space (X, ρ) with a metric ρ, $n > 1$. If it is multicentered then*

$$\pi(\mathbb{S}_n) < \frac{a}{n-1}.$$

Theorem 9. *Let's assume we have an O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ in a metric space (X, ρ) with a metric ρ, $n > 1$. If we have that*

$$\Phi(\mathbb{S}_n) < \frac{a}{n-1},$$

then this O-spheroid is multicentered.

Theorem 10. *Either all open and closed O-spheroids in arbitrary metric space (X, ρ) with a metric ρ, or their borders, are bounded sets.*

Remark 11. All closed O-spheroids in any Euclidean metric space $(\mathbb{R}^m, \rho)$ with a standard metric ρ are compact sets.

Definition 12. Metric space (X, ρ) with a metric ρ is called *convex*, if next conditions are satisfied:
1) X is a linear vector space;
2) $\forall \{x, y, z\} \subset X \, \forall \alpha \in [0; 1]$ we get:

$$\rho(\alpha x + (1-\alpha)y, z) \leq \alpha\rho(x, z) + (1-\alpha)\rho(y, z).$$

Theorem 13. *If (X, ρ) is a convex metric space with a metric ρ, then $\forall \{x_1, \ldots, x_n\} \subset X \, \forall a > 0$ open O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a connected set.*

Remark 14. All O-spheroids in linear normed spaces are connected sets.

Theorem 15. *If (X, ρ) is a convex metric space with a metric ρ, then $\forall \{x_1, \ldots, x_n\} \subset X \, \forall a > 0$ open O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a connected set.*

Theorem 16. *Let's assume that $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a non-empty O-spheroid in a convex metric space (X, ρ) with a metric ρ. Then its border is equal to its boundary.*

Definition 17 ([7, c. 236])**.** *Fermat–Torricelli point* for fixed points $\{x_1, \ldots, x_n\}$ is such point $\overline{x} \in X$, that $\forall x \in X$:

$$\sum_{k=1}^{n} \rho(\overline{x}, x_k) \leq \sum_{k=1}^{n} \rho(x, x_k).$$

Definition 18. *Voronoi radius* of O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ we call number

$$R(\mathbb{S}_n) := \sup_{x \in \mathbb{S}_n} \inf_{y \in \partial \mathbb{S}_n} \rho(x, y).$$

Theorem 19. *Let's assume that $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a non-empty O-spheroid in any Euclidean metric space $(\mathbb{R}^m, \rho)$ with a standard metric ρ, meanwhile $\overline{x}$ is a Fermat–Torricelli point for its foci. Then next inequality is correct:*

$$\frac{a - \sum\limits_{k=1}^{n} \rho(\overline{x}, x_k)}{n} \leq R(\mathbb{S}_n).$$

References

[1] Радемахер Г., Тёплиц О., Числа и фигуры: Опыты математического мышления. Выпуск 10, Библиотека математического кружка, Москва, 1962, no. 3, 263 с.

[2] Дороговцев А.Я., Математичний аналіз: Підручник, Частина 2, Либідь, Київ, 1994, 304 с.

[3] Ермаков В., Эллипс. Полная элементарная теория, Вестник опытной физики и элементарной математики, 1891, вып. 110, 21–26.

[4] Яглом И.М., Болтянский В.Г., Выпуклые фигуры. Выпуск 4, Библиотека математического кружка, 1951, Москва, Ленинград, 343 с.

[5] Пуанкаре А. Лекции по небесной механике, Наука, Москва, 1965, 571 с.

[6] Березанский Ю.М., Ус Г.Ф., Шефтель З.Г., Функциональный анализ. Курс лекций: Учеб. пособие, Выща шк., Киев, 1990, 600 с.

[7] Boltyanski V., Martini H., Soltan V., Combinatorial optimization, Volume 4: Geometric methods and optimization problems, Springer-Science+Business Media, B.V., Dordrecht, 1999, 429 pages.

[8] Ochkov V., Nori M., Borovinskaya E., Reschetilowski W., A new ellipse or math porcelain service, *Symmetry* **2019** (2019), no. 11, 184, 18 pages.

[9] Ochkov V., Vasileva I., Borovinskaya E., Reschetilowski W., Approximation of generalized ovals and lemniscates towards geometric modeling, *Mathematics* **2021** (2021), no. 9, 3325, 17 pages.

[10] Overfelt P., Superspheroid geometries for radome analysis, Technical Information Department, Naval Air Warfare Center Weapons Division, China Lake, CA, USA, 1994, 40 pages.

[11] Sekino J., n-ellipses and the minimum distance sum problem, *Amer. Math. Monthly* **106** (1999), no. 3, 193–202.

Infinitesimal deformations of surfaces of negative Gaussian curvature with a stationary Ricci tensor

T. Podousova
(Odessa National Academy of Civil Engineering and Architecture, Odessa, Ukraine)
E-mail: tatyana1985top@gmail.com

N. Vashpanova
(Odessa National Academy of Civil Engineering and Architecture, Odessa, Ukraine)
E-mail: vasha.nina@ukr.net

In [1] it was proved that every simply connected surface $S \in C^4$ non-zero Gaussian and middle of curvatures admits infinitely small (in.sm.) first-order deformations with a stationary Ricci tensor whose tensor fields have the representations

$$T^{\alpha\beta} = \varphi g^{\alpha\beta}, \ T^k = \varphi_\alpha d^{\alpha k} + \mu_\alpha c^{\alpha\beta} d^k_\beta,$$

where functions $\mu\left(x^1, x^2\right)$ and $\varphi\left(x^1, x^2\right)$ of class C^3 satisfy the following second-order partial differential equation:

$$\left(d^{\alpha\beta}\varphi_\alpha\right)_{,\beta} + 2H\varphi = \mu_{\alpha,k} c^{\alpha\beta} d^k_\beta + \mu_\alpha c^{\alpha\beta}\left(d^k_\beta\right)_{,k}. \tag{1}$$

Let S be a surface of negative Gaussian curvature. Then (1) is an equation of hyperbolic type, which in asymptotic lines takes the form

$$\varphi_{12} + d\varphi_1 + l\varphi_2 + c\varphi = f(\mu) \tag{2}$$

where d, l, c are known functions of the point S, $\mu\left(x^1, x^2\right)$ is predefined function.

For equation (2), consider the Darboux problem: We will look for such an integral that takes certain values on the characteristics $x^1 = x^1_0, \ x^2 = x^2_0$; $\varphi\left(x^1, x^2_0\right) = \lambda(x^1)$, $\varphi\left(x^1_0, x^2\right) = \tau(x^2)$.

Then each pair of functions will $\lambda(x^1), \tau(x^2)$ match the only solution $\varphi(x^1, x^2)$ equation (2) with known right side [2].

Fair

Theorem 1. *Every simply connected surface of negative Gaussian curvature of the class C^4 and without umbilical points admits ain.sm.deformations of the first order with preservation of the Ricci tensor. In this case, the strain tensors are expressed in terms of a preassigned function of two variables and two arbitrary functions of the class C^3, each from one variable.*

It should be noted that many phenomena in mechanics, physics, and biology are reduced to the study of hyperbolic equations. To describe these phenomena completely for hyperbolic equations, the Darboux problem is posed.

References

[1] T. Podousova, N. Vashpanova. Deformations of surfaces from stationary Ricci Tensor. *Mechanics and Mathematical methods*, 2(2):51–62, 2020. https://doi.org/10.31650/2618-0650-2020-2-2-51-62

[2] A. V. Bitsadze. *Some classes of partial differential equations.* M: Nauka, 1981.

Structures of optimal flows on the Boy's and Girl's surfaces

Alexandr Prishlyak
(Taras Shevchenko National University of Kyiv)
E-mail: `prishlyak@yahoo.com`

For a closed oriented surface, the Morse-Smale flows with a minimum number of fixed points (optimal ms-flow) has a single source and sink, is defined by a chord diagram, and can be embedded in R^3 [3]. For the projective plane, the optimal flow has three critical points, but it cannot even be mapped on any immersion in R^3. The simplest immersions with one triple point are Boy's and Girl's surfaces [1, 2]. Each of the surfaces has a natural stratification (cellular structure). It consists of one 0-strata, three 1-strata (A, B, C) and four 2-strata. In the Boy's surface 2-strata are set by their boundaries: A, B, C, $ABA^{-1}CAC^{-1}BCB^{-1}$. On the Girl's surface, the boundaries of 2-strata are as follows: A, B, $ABA^{-1}CB^{-1}$, $AC^{-1}C{-}1BC$.

We describe all possible structures of flows on these surfaces with respect to the homeomorphism (*isotopy*) of the surface using separatrix diagrams and methods of papers [4, 5, 6, 7].

For the flows with one isolated point and a minimum number of separatrices, there are 18 (*108*) structures per Boy's surface (with one separatrix) and 3 (*6*) structures per Girl's surface (without separatrices).

For optimal ms-flows on the surfaces as stratified sets, there are 342 (*2004*) and 534 (*1058*) flows, respectively. These flows have by 4 fixed points: 0-strata and by one point on each 1-strata.

There are 80 (*438*) and 118 (*230*) different structures for the ms-flows on the projective plane that are mapping on these surfaces. The flows have by 3 sources, 3 sinks and 5 saddles (0-strata has triple counting and points from 1-stratas have double counting).

References

[1] Werner Boy, Uber die Curvatura Integra und die Topologie der Geschlossener Flachen *Mathematische Annalen*, 57: 151 – 184, 1903.

[2] Sue Goodman, Marek Kossowski, Immersions of the projective plane with one triple point *Differential Geometry and Its Applications*, 27: 527–542, 2009.

[3] Oleksandr Kadubovskyj, Classification of Morse-Smale vector fields on 2-manifolds *Visn., Mat. Mekh., Kyiv. Univ. Im. Tarasa Shevchenka*, 14: 85–88, 2005.

[4] Alexandr Prishlyak. Complete topological invariants of Morse-Smale flows and handle decompositions of 3-manifolds *Journal of Mathematical Sciences*, 144(5): 4492–4499, 2007.

[5] Alexandr Prishlyak, Andrii Prus, Three-color graph of the Morse flow on a compact surface with boundary *Journal of Mathematical Sciences*, 249: 661–672, 2020.

[6] Alexandr Prishlyak, Maria Loseva. Optimal Morse–Smale flows with singularities on the boundary of a surface *Journal of Mathematical Sciences*, 243: 279–286, 2019.

[7] Alexandr Prishlyak, Maria Loseva. Topology of optimal flows with collective dynamics on closed orientable surfaces *Proc. Int. Geom. Cent*, 13(2): 50–67, 2020.

About solvability of the matrix equation $AX = B$ over Bezout domains

Volodymyr M. Prokip
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, L'viv, Ukraine)
E-mail: v.prokip@gmail.com

Let R be a Bezout domain with identity $e \neq 0$, i.e. R is an integral domain in which every finite generated ideal is principal. Further, let $\mathrm{R}_{m,n}$ denote the set of $m \times n$ matrices over R, and $GL(n, \mathrm{R})$ be the set of $n \times n$ invertible matrices over R. In what follows, I_n is the identity $n \times n$ matrix, $0_{m,k}$ is the zero $m \times k$ matrix, $d_i(A) \in \mathrm{R}$ is an ideal generated by the $i-$th order minors of the matrix $A \in \mathrm{R}_{m,n}$, $i = 1, 2, \ldots, \min\{m, n\}$.

Let $A \in \mathrm{R}_{m,n}$ and $B \in \mathrm{R}_{m,k}$ be nonzero matrices. Consider the nonhomogeneous matrix equation

$$AX = B, \tag{1}$$

where X is unknown matrix in $\mathrm{R}_{n,k}$. Denote by $A_B = \begin{bmatrix} A & B \end{bmatrix} \in \mathrm{R}_{m,n+k}$ the extended matrix of the linear equations (1). It is known (see [1], [3], [4], [6]) that the equation (1) over a Bezout domain R is solvable if and only if $\operatorname{rank} A = \operatorname{rank} A_B = r$ and $d_i(A) = d_i(A_B)$ for all $i = 1, 2, \ldots, r$.

The problem of solvability of the equation (1) has drawn the attention of many mathematicians (see [1]–[12] and references therein). This is explained not only by the theoretical interest to this problem ([1], [3], [4], [6], [8]–[11]), but also by the existence of numerous applied problems connected with the necessity of solution of linear matrix equations ([2], [5], [7], [12]). It may be noted, that the equation (1) over Bezout domains is important in automatic control theory [2].

1. On application of the Hermite Normal Form. In the Bezout domain R we fix a set of non-associated elements $\widetilde{\mathrm{R}}$. Every non-associated element $a \in \widetilde{\mathrm{R}}$ we associated with a complete system of residues modulo ideal (a). Let $A \in \mathrm{R}_{m,n}$ and $\operatorname{rank} A = r$. Further, we assume that the first row of the matrix A is nonzero. For the matrix A there exists $W \in GL(n, \mathrm{R})$ such that

$$AW = H_A = \begin{bmatrix} H_1 & 0_{m_1,n-1} \\ H_2 & 0_{m_2,n-2} \\ \cdots & \cdots \\ H_r & 0_{m_r,n-r} \end{bmatrix} = \begin{bmatrix} H(A) & 0_{m,n-r} \end{bmatrix}$$

is a lower block-triangular matrix, where $H(A) \in \mathrm{R}_{m,r}$, $H_1 = \begin{bmatrix} h_1 \\ * \end{bmatrix} \in \mathrm{R}_{m_1,1}$, $H_2 = \begin{bmatrix} h_{21} & h_2 \\ * & * \end{bmatrix} \in \mathrm{R}_{m_2,2}$, $\ldots$, $H_r = \begin{bmatrix} h_{r1} & \ldots & h_{r,r-1} & h_r \\ * & * & * & * \end{bmatrix} \in \mathrm{R}_{r,r}$ and $m_1 + m_2 + \cdots + m_r = m$. The elements h_i belong to the set of non-associated elements $\widetilde{\mathrm{R}}$ for all $i = 1, 2, \ldots, r$. Moreover, in the first rows $\begin{bmatrix} h_{i1} & \ldots & h_{i,i-1} & h_i \end{bmatrix}$ of the matrices H_i, $i \geq 2$, the elements h_{ij} belong to a complete system of residues modulo ideal (h_i) for all $j = 1, 2, \ldots, i-1$. The lower block-triangular matrix H_A is called the (right) Hermite normal form of the matrix A and it is uniquely defined for A (see [3]).

In this parch we propose necessary and sufficient conditions of solvability for the equation (1) over a Bezout domain in terms of the Hermite normal forms of $m \times (n + k)$ matrices $\begin{bmatrix} A & 0_{m,k} \end{bmatrix}$ and $\begin{bmatrix} A & B \end{bmatrix}$. A method for finding its solutions is also given. In what follows, we assume that the fest row of the matrix A is nonzero.

Theorem 1. *Let $A \in \mathrm{R}_{m,n}$ and $B \in \mathrm{R}_{m,k}$. The matrix equation $AX = B$ is solvable over a Bezout domain* R *if and only if the Hermite normal forms of matrices $\begin{bmatrix} A & 0_{m,k} \end{bmatrix}$ and $\begin{bmatrix} A & B \end{bmatrix}$ are coincide.*

It is easy to see that matrix equation (1) is solvable over a Bezout domain R if and only if the matrix equation $H(A)Y = B$ is solvable over R. Let $Y_0 \in \mathrm{R}_{r,k}$ be the solution of $H(A)Y = B$. Then for arbitrary matrix $P \in \mathrm{R}_{n-r,k}$ the matrix $X_P = W^{-1}\begin{bmatrix} Y_0 \\ P \end{bmatrix}$ is a general solution of equation (1). Theoretically speaking, the solution $X_0 = W^{-1}\begin{bmatrix} Y_0 \\ 0_{m-r,n} \end{bmatrix}$ of equation (1) can be written as the matrix expression $X_0 = TX_P$, where $T \in \mathrm{R}_{n,n}$. Thus, X_P is the right divisor of X_0 for an arbitrary matrix $P \in \mathrm{R}_{n-r,k}$. Given the solution X_0, we determine all possible ranks of other solutions of the equation (1), i.e. $\mathrm{rank}B \leq \mathrm{rank}X_P \leq n + \mathrm{rank}B - \mathrm{rank}A$.

2. A method of matrix transformations. In this part we apply matrix transformations for established conditions under which the equation (1) is solvable.

Let $A \in \mathrm{R}_{m,n}$ and $B \in \mathrm{R}_{m,k}$ be nonzero matrices and let $\mathrm{rank}\, A = r \geq 1$. For A there exist matrices $U \in GL(m, \mathrm{R})$ and $V \in GL(n, \mathrm{R})$ such that $UAV = \begin{bmatrix} C & 0_{r,n-r} \\ 0_{m-r,r} & 0_{m-r,n-r} \end{bmatrix}$, where $C \in \mathrm{R}_{r,r}$. It is clear that $\det C = c \neq 0$. In what follows $C^* = \mathrm{Adj}\, C$ means the classical adjoint matrix of the matrix C, i.e. $C^*C = cI_r$. Based on the above, we obtain the following theorem.

Theorem 2. *The matrix equation $AX = B$ is solvable over a Bezout domain* R *if and only if* $UB = \begin{bmatrix} D \\ 0_{m-r,k} \end{bmatrix}$, *where $D \in \mathrm{R}_{r,k}$, and $C^*D = cG$, where $G \in \mathrm{R}_{r,k}$.*

If the equation $AX = B$ is solvable, then for arbitrary matrix $Q \in \mathrm{R}_{m-r,k}$ the matrix $X_Q = U^{-1}\begin{bmatrix} G \\ Q \end{bmatrix}$ is a general solution of equation $AX = B$.

From Theorem 2 we obtain the following comment. Let $A, B \in \mathrm{R}_{m,n}$ be nonzero matrices and let $\mathrm{rank}\, A < n$. Suppose the matrix equation $AX = B$ is solvable and $X_Q \in \mathrm{R}_{n,n}$ is its general solution. Then $AX = B$ has solutions $\widetilde{X}_i \in \mathrm{R}_{n,n}$, $i = 1, 2, \ldots$, such that $X_Q = \widetilde{X}_i T_i$, where $T_i \in \mathrm{R}_{n,n}$.

Presented results above can be extended to linear nonhomogeneous equations over commutative rings of a more general algebraic nature.

References

[1] Camion P., Levy L.S., Mann H.B. Linear equations over a commutative ring. *J. Algebra.* 18(3) : 432–446, 1971.

[2] Emre E, Khargonekar P. Pole placement for linear systems over Bezout domains. *IEEE Trans Automat Control.* 29(1): 90–91, 1984.

[3] Friedland S. *Matrices.* University of Illinois at Chicago, Chicago, 2010.

[4] Hermida-Alonso J. A. On linear algebra over commutative rings. *Handbook of Algebra.* 3 : 3—61, 2003.

[5] Kaczorek T. *Polynomial and rational matrices. Applications in dynamical systems theory, communications and control engineering.* Dortrecht.: Springer, 2007.

[6] Kazimirskii P.S. Consistency conditions for the inhomogeneous system of linear equations in a noncommutative ring of principal ideals, Nauch. Zap. Lviv. Politekh. Inst., Ser. Fiz.-Mat. 30(1) : 45–51, 1955.

[7] Mulders T., Storjohann A. Certified dense linear system solving. *J. Symbol. Comput.* 37(4) : 485–510, 2004.

[8] Prokip V.M. On the solvability of a system of linear equations over the domain of principal ideals. *Ukrainian Mathematical Journal.* 66(4) : 633-638, 2014.

[9] Prokip V.M. The structure of symmetric solutions of the matrix equation $AX = B$ over a principal ideal domain. *Inter. J. Analysis.* 2017. – T. 2017.

[10] Prokip V.M. On the solvability of a system of matrix equations $AX = B$ and $BY = A$ over associative rings. *J. Math. Sciences.* 238(1) : 22–32, 2019.

[11] Uhlig F. On the matrix equation $AX = B$ with applications to the generators of a controllability matrix. *Linear Algebra Appl.* 85: 203–209, 1987.

[12] Wu Y., Yang Y. Cramer's rule over residue class rings of Bézout domains. *Linear Multilinear Algebra.* 66(6) : 1268–1276, 2018.

Regularization Method for a class of inverse problem

Nabil Saouli
(University of Badji Mokhtar Annaba)
E-mail: newnabil21@gmail.com

Fairouz Zouyed
(University of Badji Mokhtar Annaba)
E-mail: fzouyed@gmail.com

Here are the names of (almost all) predefined theorem-like environments.

Theorem 1. *For given $f \in H$ and $g \in H$ the problem*

$$\begin{cases} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(0) = g \end{cases} \tag{1}$$

has a unique solution $u \in C([0,T),H) \cap C^1((0,T),H)$ given by

$$u = e^{-tA}g + A^{-1}(I - e^{-tA})f \tag{2}$$

(JA. Goldstein, Semi-groups of linear operators and applications, Oxford university, press New York. 1985.).

Lemma 2. *For $0 < \alpha < 1$ et $p > 0$, on a les estimations suivantes :*

$$sup_{n\geqslant 1}(1 - \frac{1}{1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1}})(1+\lambda_n^2)^{\frac{-p}{2}} \leqslant max(1, T_1^{p-2}, T_1^p)max(\alpha, (\ln(\frac{1}{\sqrt{\alpha}}))^{-p}) \tag{3}$$

$$sup_{n\geqslant 1}\frac{\beta_n e^{-\lambda_n T_i}}{1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1}} \leqslant max(1, T_1^{-1})\frac{\gamma}{\sqrt{\alpha}}, i = 1,2 \tag{4}$$

$$sup_{n\geqslant 1}\frac{\beta_n}{(1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1})\lambda_n} \leqslant max(1, \lambda_1^{-2})\frac{\gamma}{\alpha}, \tag{5}$$

With

$$\gamma = \frac{1}{1 - e^{-\lambda_1(T_2 - T_1)}} \tag{6}$$

Problem 3.
Let H be a separable Hilbert space with the inner product (.;.) and the norm $\|.\|$ and let $A\colon H \to H$ be a positive self-adjoint linear operator with a compact resolvent. Consider the following final value problem:

$$\begin{cases} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(T_1) = \Psi_1 \end{cases} \tag{7}$$

where $0 < T_1 < T_2$ and Ψ_1 is a given function on H Our purpose is to identify the initial condition $u(0)$ and the unknown source f from the overspecied data $u(T_2) = \Psi_2, \Psi_2 \in H$
Hence, the inverse problem can be formulated as follows: determine f and g such that

$$\begin{cases} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(0) = g \end{cases} \tag{8}$$

from the data

$$\begin{cases} u(T_1) = \Psi_1 \\ u(T_2) = \Psi_2 \end{cases} \tag{9}$$

Corollary 4. *Let f et g the solutions of* (1) , f_α^δ *et* g_α^δ *be the modified Tikhonov approximations, Let* ψ_1^δ *and* ψ_2^δ *be the measured data at* T_1 *and* T_2 *satisfying* (9), *If the regularization parameter is chosen as* $\alpha = (\frac{\delta}{E_1})^{\frac{2}{(p_1+2)}}$ *and* $\alpha = (\frac{\delta}{E_2})^{\frac{2}{(p_2+2)}}$ *spectively then, the following error estimates hold respectively:*

$$\|f - f_\alpha^\delta\| \leqslant max(1, T_1^{p_1-1}, T_1^{p_1}) max((\frac{\delta}{E_1})^{\frac{2}{p_1+2}}, \frac{1}{(ln(\frac{E_1}{\delta})^{\frac{1}{p_1+2}})^{p_1}},) + \gamma max(1, T_1^{-1})(\frac{\delta}{E_1})^{\frac{p_1+1}{p_1+2}} E_1^{\frac{p_1}{p_1+2}} \tag{10}$$

$$\|g - g_\alpha^\delta\| \leqslant max(1, T_1^{p_2-1}, T_1^{p_2}) max((\frac{\delta}{E_2})^{\frac{2}{p_2+2}}, \frac{1}{(ln(\frac{E_2}{\delta})^{\frac{1}{p_2+2}})^{p_2}},) + \gamma max(1, T_1^{-1})(\frac{\delta}{E_2})^{\frac{p_2}{p_2+2}} E_2^{\frac{2+p_2}{p_2+2}} \tag{11}$$

References

[1] JA. Goldstein, Semigroups of linear operators and applications, Oxford university, press New York. 1985.

[2] C.L. Fu, Simplified Tikhonov and Fourier regularization methods on a general sideways parabolic equation, J. Comput. Appl. Math. 167 (2004), 449-463.

Broadening of some vanishing theorems of global character about holomorphically projective mappings of Kahlerian spaces to the noncompact but complete ones.

Helena Sinyukova
(State institution «South Ukrainian National Pedagogical University named after K. D. Ushinsky»)
E-mail: `olachepok@ukr.net`

The generalized Bochner technique (see, for example, [1]) allows to broad to the noncompact but compete Kahlerian spaces some well-known theorems of holomorphically projective unique definability that have been proved previously for the compact ones (see, for example, [2]). Thus, the next theorems are true.

Theorem 1. *Complete connected noncompact Kahlerian C^r-space K^n $(n > 2,\ r > 4)$ with positive defined metric tensor and the Einstein tensor that doesn't equal to zero, that satisfies the recurrent conditions*

$$T^{(\alpha\beta)}_{ijkl,mh} g^{mj} g^{hl} E^{ik}_{..} = \frac{1}{n} T^{(\alpha\beta)}_{\gamma h} \left(\delta^{\gamma}_{\mu} g_{\nu m} + F^{\gamma}_{\mu} F_{\nu m}\right) T^{(\mu\nu)}_{ijkl} g^{mj} g^{hl} E^{ik}_{..} + T^{(\alpha\beta)}_{ijkl} W^{ijkl} + T^{(\alpha\beta)}_{ijkl,m} W^{ijklm},$$

where

$$T^{\alpha\beta}_{ijkl} = n\delta^{\alpha}_{(i} R^{\beta}_{j)kl} + g_{l(i} T^{\alpha\beta}_{j)k} - g_{k(i} T^{\alpha\beta}_{j)l} - F_{l(i} F^{\gamma}_{j)} T^{\alpha\beta}_{\gamma k} + F_{k(i} F^{\gamma}_{j)} T^{\alpha\beta}_{\gamma l},$$

$$T^{\alpha\beta}_{\gamma l} = \delta^{\alpha}_{i} R^{\beta}_{k} - R^{\alpha}_{ik}{}^{\beta}_{.}$$

F^{i}_{j} – components of tensor of complex structure, R_{ij} — components of Ricci tensor, E_{ik} – components of Einstein tensor of the space K^n; W^{ijkl}, W^{ijklm} — components of some contravariant tensors, "," denotes the corresponding covariant differentiation, doesn't admit non-trivial (different from affine) holomorphically projective mappings on the whole.

Theorem 2. *Complete connected noncompact Kahlerian C^r-space K^n $(n > 2,\ r > 4)$ with positive defined metric tensor and the Einstein tensor that doesn't equal to zero, that satisfies the recurrent conditions*

$$P^{(\alpha\beta)}_{il,kh} g^{hi} E^{kl}_{..} = P^{(\alpha\beta)}_{il,k} S^{ilk} + P^{(\alpha\beta)}_{il} S^{il}, \tag{1}$$

where

$$P^{\alpha\beta}_{il} = \delta^{\beta}_{i} R^{\alpha}_{.l} - \delta^{\beta}_{l} R^{\alpha}_{.i},$$

R_{ij} — components of Ricci tensor, E_{ij} — components of Einstein tensor of the space K^n; S^{ilk}, S^{il} — components of some contravariant tensor, "," denotes the corresponding covariant differentiation, doesn't admit non-trivial (different from affine) holomorphically projective mappings on the whole.

Recurrent conditions (1) may also be transformed to the more general form.

Examples of Kahlerian spaces of considered types are known.

References

[1] Pigola S., Rigoli M., Setti A.G. *Vanishing in finitness results in geometric analysis.* in *A Generalization of the Bochner Technique.*, Berlin: Birkhauser Varlag AG, 2008

[2] Sinyukova, H.N. On some classes of holomorphically-projectively uniquely defined Kahlerian spaces on the whole, *Proc. Intern. Geom. Center*, 3(4) : 15–24, 2010.

The weight of T_0-topologies on n-element set that consistent with close to the discrete topology on $(n-1)$-element set

Anna Skryabina
(Department of Universal Mathematics, Zaporizhzhya National University, Zhukovskogo str. 66, building 1, office 21-A, Zaporizhzhya, 69600, Ukraine)
E-mail: anna_29_95@ukr.net

Polina Stegantseva
(Department of Universal Mathematics, Zaporizhzhya National University. Zhukovskogo str. 66, building 1, office 21-A, Zaporizhzhya, 69600, Ukraine)
E-mail: stegpol@gmail.com

The topologies on an n-element set with weight $k > 2^{n-1}$ (k is the number of the elements of the topology) are called close to the discrete topology. In [1] all T_0-topologies have been listed using the topology vector, an ordered set of the nonnegative integers $(\alpha_1, \alpha_2, ..., \alpha_n)$, α_i is one less than the number of the elements in the minimum neighborhood M_i of the element x_i. In [2] T_0-topologies on an n-element set with the vectors $(0, ..., 0, \alpha_{n-1}, \alpha_n)$ and $(0, ..., 0, 1, 1, \alpha_n)$ in the case $M_{n-1} \cap M_{n-2} = \emptyset$ have been investigated. These T_0-topologies are consistent with close to the discrete topology on $(n-1)$-element set with the vectors $(0, ..., 0, \alpha_{n-1})$ and the vector $(0, ..., 0, 1, 1)$ in the case $M_{n-1} \cap M_{n-2} = \emptyset$. The question about T_0-topologies which are consistent with close to the discrete topology on $(n-1)$-element set with vectors $(\underbrace{0, ..., 0}_{k}, 1, ..., 1)$, $1 \le k \le n-3$, where all $n-1-k$ two-element minimum neighborhoods have only one common point, remains unresolved. This work we found the weight of these T_0-topologies.

So, the vector of T_0-topologies has the form: $(\underbrace{0, ..., 0}_{k}, \underbrace{1, ..., 1}_{n-k-1}, \alpha_n)$, $1 \le k \le n-3$, $2 \le \alpha_n \le n-1$ and $\bigcap_{m=k+1}^{n-1} M_m = \{x_1\}$. The following cases are possible for the minimum neighborhood M_n of the element x_n:

1. $\bigcap_{m=k+1}^{n-1} M_m \cap M_n = \{x_1\}$, so $M_n = \{x_1, ..., x_d, \underbrace{x_{n-(\alpha_n-d)}, ..., x_{n-1}}_{\alpha_n-d}, x_n\}$. The general formula for the weight has the form $|\tau| = 2^{n-2} + 2^{k-1} + 2^{k-d} + 2^{k-d} \cdot (2^{n-k-(\alpha_n-d+1)} - 1)$.
2. $\bigcap_{m=k+1}^{n-1} M_m \cap M_n = \emptyset$. The general formula for the weight has the form $|\tau| = 2^{n-2} + 2^{k-1} + 2^{k-\alpha_n} + 2^{k-(\alpha_n+1)} \cdot (2^{n-k-1} - 1)$.

References

[1] Bashova N.P. Velichko I.G., Stegantseva P.G. Enumeration of topologies close to discrete on finite sets. *Izvestiya vuzov. Matematika*, 11 : 23–31, 2015.

[2] Stegantseva P.G., Skryabina A.V. Topologies on the n-element set consistent with topologies close to the discrete on an $(n-1)$-element set *Ukrainian Mathematical Journal*, 2(73) : 276–288, 2021.

On ternary assymetric medial top-quasigroups

Fedir Sokhatsky
(Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine)
E-mail: fmsokha@ukr.net

Iryna Fryz
(Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine)
E-mail: iryna.fryz@ukr.net

Let Q be an m element set. A ternary operation f defined on Q is called *invertible* and the pair $(Q; f)$ is a *quasigroup* of the order m, if for every a, b of Q the terms $f(x, a, b)$, $f(a, x, b)$, $f(a, b, x)$ define permutations of Q. To each ternary quasigroup $(Q; f)$ of the order m there corresponds a Latin cube of order m, i.e., a 3-dimensional array on m distinct symbols from Q, each of which occurs exactly once in any line of the array.

A triplet (f_1, f_2, f_3) of ternary operations is called *orthogonal* [1], if for all $a_1, a_2, a_3 \in Q$ the system

$$\begin{cases} f_1(x_1, x_2, x_3) = a_1, \\ f_2(x_1, x_2, x_3) = a_2, \\ f_3(x_1, x_2, x_3) = a_3 \end{cases}$$

has a unique solution, i.e., superimposition of the corresponding cubes gives a cube such that every triplet of elements of Q appears exactly once in it.

Geometric interpretation of orthogonality is its relationships with geometric nets. This application is well-studied for binary operations and the respective k-nets, projective and affine planes (see for example [2], [3]). Relationships between t-tuples of orthogonal n-ary quasigroups of order m and (t, m, n)-nets were studied in [4], [5], [6]. The respective nets have the same combinatorial and algebraic properties.

For every permutation $\sigma \in S_4$ a *σ-parastrophe* ${}^{\sigma}f$ of an invertible ternary operation f is defined by

$${}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} :\Longleftrightarrow f(x_1, x_2, x_3) = x_4.$$

In particular, a σ-parastrophe is called:

- an *i-th division* if $\sigma = (i4)$ for $i = 1, 2, 3$;
- *principal* if $4\sigma = 4$.

Therefore, each ternary operation has at most $4! = 24$ parastrophes; among them $3! = 6$ principal parastrophes. An invertible operation and the respective quasigroup are called *assymetric* if all its parastrophes are different. A quasigroup is called *totally parastrophic orthogonal* (*top-quasigroup*), if each triplet of its different parastrophes are orthogonal. Binary assymetric top-quasigroups were studied in [7], for ternary case the following statements are true.

Theorem 1 ([8]). *A quasigroup $(Q; f)$ is medial if and only if there exists an abelian group $(Q; +)$ such that*

$$f(x_1, x_2, x_3) = \varphi_1 x_1 + \varphi_2 x_2 + \varphi_3 x_3 + a, \tag{1}$$

where φ_1, φ_2, φ_3 are pairwise commuting automorphisms of $(Q; +)$ and $a \in Q$.

Theorem 2. *Let* $(Q;f)$ *be a medial ternary quasigroup* $(Q;f)$ *with* (1) *and* $\tau_1,\ \tau_2,\ \tau_3 \in S_4$. *The parastrophes* ${}^{\tau_1}f,\ {}^{\tau_2}f,\ {}^{\tau_3}f$ *are orthogonal if and only if the determinant*

$$\begin{vmatrix} \varphi_{1\tau_1} & \varphi_{2\tau_1} & \varphi_{3\tau_1} \\ \varphi_{1\tau_2} & \varphi_{2\tau_2} & \varphi_{3\tau_2} \\ \varphi_{1\tau_3} & \varphi_{2\tau_3} & \varphi_{3\tau_3} \end{vmatrix}$$

is an automorphism of the group $(Q;+)$, *where* $\varphi_4 := J$ *and* $J(x) := -x$.

Note, that the pairwise commuting automorphisms $\varphi_1,\ \varphi_2,\ \varphi_3,\ J$ generate a commutative subring K of the ring $End(Q;+)$. Let $\vec{\nu} := (\nu_1,\nu_2,\nu_3)$ be a triplet of injections of the set $\{1,2,3\}$ into the set $\{1,2,3,4\}$. The polynomial

$$d_{\vec{\nu}}(\gamma_1,\gamma_2,\gamma_3,\gamma_4) := \begin{vmatrix} \gamma_{1\nu_1} & \gamma_{2\nu_1} & \gamma_{3\nu_1} \\ \gamma_{1\nu_2} & \gamma_{2\nu_2} & \gamma_{3\nu_2} \\ \gamma_{1\nu_3} & \gamma_{2\nu_3} & \gamma_{3\nu_3} \end{vmatrix}$$

over the commutative ring K will be called *invertible-valued* over a set $H \subseteq K$, if all its values are automorphisms of the group $(Q;+)$ when the variables $\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4$ take their values in H.

Theorem 3. *A ternary medial quasigroup* $(Q;f)$ *with* (1) *is a top-quasigroup if and only if each polynomial* $d_{\vec{\nu}}$ *is invertible-valued over the set* $\{\varphi_1,\varphi_2,\varphi_3,\varphi_4\}$, *where* $\varphi_4 := J$.

Theorem 4 ([9]). *A ternary medial assymetric top-quasigroup over a cyclic group of the order* m *exists if and only if the least prime factor of* m *is greater than 19.*

References

[1] Galina B. Belyavskaya, Gary L. Mullen. Orthogonal hypercubes and n-ary operations, *Quasigroups and Related System*, 13: 73–76, 2005.

[2] V.D. Belousov. *Configurations in algebraic webs.* Kishinev: Stiintsa, 1979. (in Russian).

[3] A.D. Keedwell, J. Dénes. *Latin Squares and their Applications.* Amsterdam: North Holland, 2015.

[4] C.F. Laywine, G.L. Mullen and G. Whittle. D-Dimensional hypercubes and the Euler and MacNeish conjectures. *Monatsh. Math.*, 111: 223–238, 1995.

[5] Maks A. Akivis, Vladislav V. Goldberg. Algebraic aspects of web geometry. *Commentationes Mathematicae Universitatis Carolinae*, 41(2): 205—236, 2000.

[6] F.M. Sokhatsky. About multiary webs. *International Conference "Morse theory and its applications" dedicated to the memory and 70th anniversary of Volodymyr Vasylyovych Sharko*, September 25-28, 2019: 47–48, 2019.

[7] G.B. Belyavskaya, T.V. Popovich. Totally conjugate orthogonal quasigroups and complete graphs. *Fundamentalnaya i prikladnaya matematika*, 13 (8): 17–26, 2010. (Russian)

[8] V.D. Belousov. *n-ary quasigroups.* Chishinau: Stiintsa, 1972. (in Russian)

[9] F. Sokhatsky, Ie. Pirus. About parastrophically orthogonal quasigroups. *Book of extended abstracts of the International Mathematical Conference on Quasigroups and Loops "Loops'15"*, 28 June - 04 July 2015: 46–47, 2015.

Extension theorems for holomorphic bundles on complex manifolds with boundary

Andrei Teleman
(Aix Marseille Univ, CNRS, I2M, Marseille, France)
E-mail: andrei.teleman@univ-amu.fr

We begin with the following important result due to Donaldson [Do] for Kähler, and Xi [Xi] for general Hermitian complex manifolds with boundary:

Theorem 1. *Let $\bar{X}$ be a compact complex manifold with non-empty boundary $\partial\bar{X}$, g be a Hermitian metric on $\bar{X}$ and $\mathcal{E}$ be a holomorphic bundle on $\bar{X}$. Let h be a Hermitian metric on the restriction $\mathcal{E}|_{\partial X}$. There exists a unique Hermitian metric H on $\mathcal{E}$ satisfying the conditions*

$$\Lambda_g F_H = 0,\ \ H|_{\partial X} = h,$$

where $F_H \in A^2(\bar{X}, \mathrm{End}(\mathcal{E}))$ denotes the curvature of the Chern connection associated with H.

Note that the map $H \mapsto \Lambda_g F_H$ is a non-linear second order elliptic differential operator, so the system $\Lambda_g F_H = 0$, $H|_{\partial\bar{X}} = h$ can be viewed as a non-linear Dirichlet problem. The theorem of Donaldson and Xi states that this non-linear Dirichlet problem is always uniquely sovable.

Note also that the analogue statement for closed manifolds (i.e. in the case $\partial\bar{X} = \emptyset$) does not hold. Indeed, the classical Kobayashi-Hitchin correspondence states that, for a holmorphic bundle $\mathcal{E}$ on a closed Hermitian manifold (X, g), the equation $\Lambda_g F_H = 0$ is solvable if and only if $\deg_g(\mathcal{E}) = 0$ (which is a topological condition if g is Kählerian) and $\mathcal{E}$ is polystable with respect to g (see [LT]).

Recall that a unitary connection ∇ on a Hermitian differentiable bundle (E, H) on $\bar{X}$ is called Hermitian Yang-Mills if $\Lambda_g F_\nabla = 0$, $F_\nabla^{02} = 0$. In the classical case $\dim_{\mathbb{C}}(X) = 2$ – which plays a fundamental role in Donaldson theory – these conditions are equivalent to the anti-self-duality condition $F_\nabla^+ = 0$.

In [Do] Donaldson shows that Theorem 1 has important geometric consequences:

Corollary 2. *Let $\bar{X}$ be a compact complex manifold with non-empty boundary, g be a Hermitian metric on $\bar{X}$ and (E, H) be a Hermitian differentiable bundle on $\bar{X}$. There exists a natural bijection between:*

(1) the moduli space of pairs $(\mathcal{E}, \theta)$ consisting of a holomorphic structure $\mathcal{E}$ on E and a differentiable trivialization θ of $E|_{\partial\bar{X}}$,

(2) the moduli space of pairs (∇, τ) consisting of a Hermitian Yang-Mills connection on (E, H) and a differentiable unitary trivialization τ of $E|_{\partial\bar{X}}$.

In other words, the moduli space of boundary framed holomorphic structures on E can be identified with the moduli space of boundary framed Hermitian Yang-Mills connection on (E, H).

In the special case when $\bar{X}$ is the closure of a strictly pseudoconvex domain (with smooth boundary) in $\mathbb{C}^n$, Donaldson states the following result which gives an interesting geometric interpretation of the quotient $\mathcal{C}^\infty(\partial\bar{X}, \mathrm{GL}(r, \mathbb{C}))/\mathcal{O}^\infty(\bar{X}, \mathrm{GL}(r, \mathbb{C}))$ of the group of smooth maps $\partial\bar{X} \to \mathrm{GL}(r, \mathbb{C})$ by the subgroup formed by those such maps which extend smoothly and formally holomorphically to $\bar{X}$:

Corollary 3. *Let $\mathcal{O}^\infty(\bar{X}, \mathrm{GL}(r, \mathbb{C}))$ be the group of smooth, formally holomorphic $\mathrm{GL}(r, \mathbb{C})$-valued maps on $\bar{X}$, identified with a subgroup of $\mathcal{C}^\infty(\partial\bar{X}, \mathrm{GL}(r, \mathbb{C}))$ via the restriction map.*

There exists a natural bijection between the moduli space of boundary framed Hermitian Yang-Mills connections on the trivial $\mathrm{U}(r)$-bundle on $\bar{X}$ and the quotient $\mathcal{C}^\infty(\partial\bar{X}, \mathrm{GL}(r, \mathbb{C}))/\mathcal{O}^\infty(\bar{X}, \mathrm{GL}(r, \mathbb{C}))$.

The idea of proof: Taking into account Corollary 2, it suffices to construct a bijection between the quotient $\mathcal{C}^\infty(\partial\bar{X}, \mathrm{GL}(r,\mathbb{C}))/\mathcal{O}^\infty(\bar{X}, \mathrm{GL}(r,\mathbb{C}))$ and the moduli space of boundary framed holomorphic structures on the trivial differentiable bundle $\bar{X}\times\mathbb{C}^r$. The construction is very natural: one maps the congruence class $[f]$ of a smooth map $f:\partial\bar{X}\to\mathrm{GL}(r,\mathbb{C})$ to the gauge class of the pair (the trivial holomorphic structure on $\bar{X}\times\mathbb{C}^r, f$). The main difficulty is to prove the surjectivity of the map obtained in this way. This follows from the following existence result:

Proposition 4. *Let $\bar{X}$ be the closure of a strictly pseudoconvex domain (with smooth boundary) in $\mathbb{C}^n$ and $\mathcal{E}$ be a smooth, topologically trivial holomorphic bundle on $\bar{X}$. Then $\mathcal{E}$ admits a global smooth trivialization on $\bar{X}$ which is holomorphic on X.*

The statement follows using Grauert's classification theorem for bundles on Stein manifolds and the following extension theorem, which is proved in [Do] only for $n=2$:

Proposition 5. *Let $\bar{X}$ be the closure of a relatively compact strictly pseudoconvex domain (with smooth boundary) in $\mathbb{C}^n$ and $\mathcal{E}$ be a smooth, topologically trivial holomorphic bundle on $\bar{X}$. Then $\mathcal{E}$ extends holomorphically to an open neighborhood U of $\bar{X}$ in $\mathbb{C}^n$.*

In my talk I will explain the idea of proof of the following general extension theorem (see [T]):

Theorem 6. *Let M be a complex manifold, $X\subset M$ an open submanifold of M whose closure $\bar{X}$ has smooth, strictly pseudoconvex boundary in M. Let G be a complex Lie group, $\pi:Q\to M$ a differentiable principal G-bundle on M and J a holomorphic structure on the restriction $\bar{P}:-Q|_{\bar{X}}$.*

There exists an open neighborhood M' of $\bar{X}$ in M and a holomorphic structure J' on $Q|_{M'}$ which extends J.

The proof uses methods and techniques introduced in [HiNa] and [Ca1].

In the special case when $M=\mathbb{C}^n$ and $G=\mathrm{GL}(r,\mathbb{C})$ one obtains as corollary Proposition 5 (and hence Corollary 3) in full generality. Moreover, one also obtains the following generalization of this corollary:

Theorem 7. *Let $G=K^{\mathbb{C}}$ be the complexification of a compact Lie group K, $\bar{X}$ be a compact Stein manifold with boundary and g be a Hermitian metric g on $\bar{X}$. The moduli space of boundary framed Hermitian Yang-Mills connections on the trivial K-bundle on $(\bar{X},g)$ can be identified with the quotient $\mathcal{C}^\infty(\partial\bar{X},G)/\mathcal{O}^\infty(\bar{X},G)$.*

References

[Ca1] D. Catlin, A Newlander-Nirenberg theorem for manifolds with boundary, Mich. Math. J., 35 (1988), 233-240.

[Do] S. Donaldson, Boundary value problems for Yang-Mills fields, Journal of Geometry and Physics 8 (1992) 89-122.

[HiNa] C. Hill, M. Nacinovich, A collar neighborhood theorem for a complex manifold, Rendiconti del Seminario Matematico della Università di Padova, tome 91 (1994), 23-30.

[Hö] L. Hörmander, Differential Operators of Principal Type, Math. Annalen 140, (1960) 124-146.

[Le] H. Lewy, "An example of a smooth linear partial differential equation without solution", Annals of Mathematics, Vol. 66, No. 1 (1957), 155–158.

[LT] M. Lübke, A. Teleman, The Kobayashi-Hitchin correspondence, World Scientific Publishing Co. (1995).

[T] A; Teleman, Holomorphic bundles on complex manifolds with boundary, arXiv:2203.10818 [math.CV].

[Xi] Z. Xi, Hermitian–Einstein metrics on holomorphic vector bundles over Hermitian manifolds, Journal of Geometry and Physics 53 (2005) 315-335.

Recent progress in Iwasawa theory of knots and links

Jun Ueki
(Department of Mathematics, Ochanomizu University, Tokyo, Japan)
E-mail: uekijun46@gmail.com

We briefly survey joint works with Ryoto Tange, Hyuga Yoshizaki, and Sohei Tateno.

Twisted Iwasawa invariants of knots [1]. Let K be a knot in S^3 with $\pi_K = \pi_1(S^3 - K)$ and let $X_n \to X = S^3 - K$ denote the $\mathbb{Z}/n\mathbb{Z}$-cover for each $n \in \mathbb{Z}_{>0}$. Let p be a prime number and let $m \in \mathbb{Z}$ with $p \nmid m$. Let $\rho : \pi_K \to \mathrm{GL}_N(O_\mathfrak{p})$ be a representation over a finite extension $O_\mathfrak{p}$ of the p-adic number field $\mathbb{Q}_p$ and let $\Delta_\rho(t)$ denote the twisted Alexander polynomial. Then we have the following.

Theorem 1. *Let (K, p, m, ρ) be as above. Then there exists some $\lambda, \mu, \nu \in \mathbb{Z}$ such that for any $n \gg 0$, $|H_1(X_{mp^n}, \rho)| = p^{\lambda n + \mu p^n + \nu}$ holds.*

For each (p, K, ρ), there exists some m such that $\lambda = \deg\Delta_\rho(t)$. Hence for each K, there exists some (p, ρ, m) such that λ coincides with the genus of K.

For each (p, K, ρ), the set of μ's and λ's determines if $\Delta_\rho(t)$ is monic in $O_\mathfrak{p}[t]$.

Example 2. (1) The λ's of the lifts $\rho^{\pm}_{\mathrm{hol}} : \pi_K \to \mathrm{SL}_2(O)$ of the holonomy representation of the figure eight knot $K = 4_1$.

(2) For any SL_2-representations of the twist knots $J(2, 2k)$ $(k \in \mathbb{Z})$, we have $\mu = 0$. We may expect that if $k \neq 0, 1$, then there exists some ρ of $J(2, 2k)$ with $\mu > 0$.

Weber's class number problem for knots [2]. Weber's class number problem for number fields is unsolved for more than 200 years. Yoshizaki [3] recently pointed out that the sequence of the class numbers converges in the ring of p-adic integers $\mathbb{Z}_p$. In the knot theory side, we obtain the following.

Theorem 3. *Let K be a knot in S^3 and let p be a prime number. Then the sizes of the p-torsion subgroups of $H_1(X_{p^n}; \mathbb{Z})$ converges in $\mathbb{Z}_p$. The limit value is given by the roots of unity that are close to the roots of the Alexander polynomial $\Delta_K(t)$.*

Example 4. The limit values for the torus knot $T_{a,b}$ $(a, b \in \mathbb{Z}$; coprime$)$ and the twist knot $J(2, 2k)$ $(k \in \mathbb{Z})$.

Iwasawa invariants of the ${\mathbb{Z}_p}^d$-covers of links [4]. Cuoco–Monsky gave a variant of the Iwasawa class number formula for ${\mathbb{Z}_p}^d$-extension of number fields and pointed out the existence of the term $O(1)$. In our side, we have the following.

Theorem 5. *Let L be a d-component link in a rational homology 3-sphere M and let $Y_n \to X = M - L$ denote the $\mathbb{Z}/n\mathbb{Z}^d$-cover. Then there exists some λ, μ such that the size of p-torsion subgroup of $H_1(Y_n, \mathbb{Z})$ is given by $p^{p^{(d-1)n}(\mu p^n + \lambda n + O(1))}$, where $O(1)$ is the Bachmann–Landau notation. If M is an integral homology 3-sphere, then the ${\mathbb{Z}_p}^d$-cover is Greenberg, namely, $O(1)$ is a constant.*

Example 6. The values μ, λ, and $O(1)$ of Solomon's link 4^2_1 and the twisted Whitehead link W_{2k-1} $(k \in \mathbb{Z})$. We have a link with $O(1) \neq 0$ and a link with any $\mu \in \mathbb{Z}_{\geq 0}$.

References

[1] Ryoto Tange and Jun Ueki. *Twisted Iwasawa invariants of knots*, preprint, 2022. arXiv:2203.03239
[2] Jun Ueki and Hyuga Yoshizaki. *On Weber's class number problem for knots*, preprint, 2022.
[3] Hyuga Yoshizaki, *A New Continued Fraction Expansion and Weber's Class Number Problem*, preprint, 2020. arXiv: 2010.06399
[4] Sohei Tateno and Jun Ueki. *Iwasawa invariants of the $\mathbb{Z}_p^d$-covers of links*, preprint, 2022.

Про тип грассманового образу поверхонь з плоскою нормальною зв'яністю простору Мінковського

Марина Гречнєва
(Запоріжжя, Запорізький національний університет)
E-mail: grechnevamarina@gmail.com

Поліна Стєганцева
(Запоріжжя, Запорізький національний університет)
E-mail: stegpol@gmail.com

Поверхня V^2 класу $C^k, k > 1$ у просторі Мінковського ${}^1R_4$ називається *просторовоподібною (часоподібною, ізотропною)*, якщо дотична площина до неї в кожній точці є просторовоподібною (часоподібною, ізотропною). Будемо розглядати такі двовимірні поверхні простору ${}^1R_4$ або такі області на цих поверхнях, у яких тип дотичної площини в кожній точці один і той самий. При грассмановому відображенні поверхні V^2 в грассманів многовид $PG(2,4)$ отримаємо *грассмановий образ* поверхні V^2. Грассманів образ просторовоподібної (часоподібної) двовимірної поверхні простору ${}^1R_4$ є двовимірним підмноговидом многовиду часоподібних (просторовоподібних) площин [2]. Індукована метрика грассманового образу може бути знаковизначеною, знаконевизначеною або виродженою, а значить грассманів образ може бути двовимірною просторовоподібною, часоподібною або ізотропною поверхнею. З'ясуємо питання про тип грассманового образу поверхонь з плоскою нормальною зв'язністю.

Поняття плоскої нормальної зв'язності підмноговиду риманового многовиду було введено Е.Картаном [1]. Підмноговиди з плоскою нормальною зв'язністю є підмноговидами з нульовим тензором скруту. Важливою властивістю поверхонь з плоскою нормальною зв'язністю є існування координатної сітки, відносно якої першу та обидві другі квадратичні форми можна одночасно звести до діагонального виду. Ця координатна сітка є сіткою ліній кривини. Поверхні з плоскою нормальною зв'язністю та їх грассманові образи у просторі Мінковського мають ще додаткові властивості:

1) якщо грассмановий образ часоподібної поверхні $V^2 \subset^1 R_4$ з плоскою нормальною зв'язністю невироджений, то він є часоподібною поверхнею;

2) невироджений грассмановий образ просторовоподібної поверхні з плоскою нормальною зв'язністю може бути або просторовоподібною, або часоподібною, або ізотропною поверхнею;

3) тип невиродженого грассманового образу гіперповерхні V^2 деякого тривимірного підпростору простору ${}^1R_4$ співпадає з типом поверхні V^2.

Література

[1] Лумисте Ю.Г., Чекмазян А.В. *Нормальная связность и подмногообразия с параллельными нормальными полями в пространстве постоянной кривизны Итоги науки и техн. Сер. Пробл. геом.*, (12): 3–30, 1981.

[2] Стеганцева П.Г., Гречнева М.А. *Грассманов образ неизотропной поверхности псевдоевклидова пространства Известия вузов. Математика.*, (2): 65-75, 2017

Про існування гедезично симетричних псевдоріманових просторів

В. Кіосак
(Одеська державна академія будівництва та архітектури, Одеса, Україна)
E-mail: kiosakv@ukr.net

Л. Кусік
(Одеський національний морський університет, Одеса, Україна)
E-mail: kusikl@ukr.net

В. Ісаєв
(Одеська державна академія будівництва та архітектури, Одеса, Україна)
E-mail: isaevv5@gmail.com

Серед робіт по геодезичним відображенням псевдоріманових просторів особливе місці займає робота 1896 року Т. Леві-Чевіти, в якій він, виходячи з рівнянь динаміки, сформулював постановку задачі та отримав основні рівняння [1]. Особливістю роботи є використання тензорних методів.

Після того, як тензорні методи дослідження зайняли домінуючі позиції в диференціальній геометрії, Г. Вейль, Л.П. Ейзенхарт, В.Ф. Каган, Г.І. Кручкович, А.С. Солодовніков та інші побудували струнку теорію геодезичних відображень псевдоріманових просторів, інваріантну відносно вибору системи координат.

Новий поштовх ця теорія отримала після робіт М.С. Синюкова, який звів задачу до дослідження лінійної системи диференціальних рівнянь [2].

Взаємно однозначна відповідність між точками псевдоріманових просторів V_n з метричним тензором g_{ij} та $\bar{V}_n$ з метричним тензором $\bar{g}_{ij}$ називається геодезичним відображенням, якщо при ній кожна геодезична лінія V_n переходить в геодезичну лінію $\bar{V}_n$.

Псевдоріманів простір V_n, в якому існує тензор $A_{i_1 i_2 \dots i_k}$ такий, що $A_{i_1 i_2 \dots i_k, j} = 0$, називають A-симетричним. Тут кома "," знак коваріантної похідної по зв'язності V_n. Геодезично A-симетричним називаємо псевдоріманів простір, в якому умова A-симетричності виконується для коваріантної похідної по зв'язності геодезично відповідного даному простору V_n псевдоріманового простору $\bar{V}_n$ [3].

Зокрема, якщо для тензора Річчі псевдоріманового простору V_n виконується умова $\nabla_k R_{ij} = 0$ (тут ∇ знак коваріантної похідної по зв'язності $\bar{V}_n$), то такій простір називаємо геодезично Річчі симетричним. Якщо ця умова виконується для тензора Рімана, то простір має назву геодезично симетричний.

Доведено, що не існує геодезично Річчі симетричних просторів відмінних від просторів Ейнштейна, а також, що не існує геодезично симетричних псевдоріманових просторів відмінних від просторів сталої кривини.

Таким чином, геодезично Річчі симетричні та геодезично симетричні простори існують лише тоді, коли вони простори Ейнштейна та простори сталої кривини відповідно.

Література

[1] T. Levi-Civita. Sulle transformationi delle equazioni dinamiche. *Ann. Mat. Milano*, Ser. 2., 24:255-300, 1896.

[2] Н.С. Синюков. *Геодезические отображения римановых пространств*. Наука, 1979.

[3] V. Kiosak, V.S. Matveev. There exist no 4-dimensional geodesically equivalent metrics with the same stress-energy tensor. *Journal of Geometry and Physics*, 78: 1–11, 2014, https://doi.org/10.1016/j.geomphys.2014.01.002

Геометричні об'єкти, інваріантні відносно квазі-геодезичних відображень псевдо-ріманових просторів з узагальнено-рекурентною афінорною структурою

Курбатова І.М.
(ОНУ, Одеса, Україна)
E-mail: irina.kurbatova27@gmail.com

Піструіл М.І.
(ОНУ, Одеса, Україна)
E-mail: margaret.pistruil@gmail.com

Досліджувалися квазі-геодезичні відображення [1] узагальнено-рекурентних просторів параболічного типу [3] (V_n, g_{ij}, F_i^h) і $(\overline{V}_n, \overline{g}_{ij})$. Основні рівняння такого відображення в сумісній за відображенням системі координат (x^i) мають вигляд [3]

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}(x)\delta_{j)}^h + \phi_{(i}(x)F_{j)}^h(x),$$
$$F_{ij} = -F_{ji}, \quad F_{ij} = g_{i\alpha}F_j^\alpha, \quad \overline{F}_{ij} = -\overline{F}_{ji}, \quad \overline{F}_{ij} = \overline{g}_{i\alpha}F_j^\alpha,$$
$$F_\alpha^h F_i^\alpha = 0$$
$$F_{(i,j)}^h = F_{(i}^h q_{j)},$$

$\overline{\Gamma}_{ij}^h, \Gamma_{ij}^h$ - компоненти об'єктів зв'язності $\overline{V}_n$ і V_n; ψ_i, φ_i - деякі ковектори; "," - знак коваріантної похідної в V_n.

Якщо диференціальні рівняння для афінора набувають вигляду $F_{(i,j)}^h = F_{(i}^h q_{j)}$, ми називаємо афінорну структуру узагальнено-рекурентною, а при $F_{i,j}^h = F_i^h q_j$ - рекурентно-параболічною.

Ми вважаємо, що узагальнено-рекурентна структура інтегровна і квазі-геодезичне відображення зберігає вектор узагальненої рекурентності [3], отже в просторі $(\overline{V}_n, \overline{g}_{ij})$ для афінора F_i^h виконуються співвідношення

$$F_{(i|j)}^h = F_{(i}^h q_{j)},$$

де "|" - знак коваріантної похідної відносно зв'язності $\overline{\Gamma}$ в V_n.

Побудовано геометричні об'єкти, інваріантні відносно квазі-геодезичного відображення узагальнено-рекурентних просторів параболічного типу, а також рекурентно-параболічних просторів. Наводиться ряд умов на ці об'єкти, що призводять до того, що узагальнено-рекурентний простір параболічного типу допускає параболічну K-структуру, для якої $F_{(i,j)}^h = 0$, а рекурентно-параболічний простір допускає келерову структуру параболічного типу.

Вивчено спеціальні типи квазі-геодезичних відображень узагальнено-рекурентних просторів, що зберігають деякі тензори внутрішнього характеру.

Література

[1] А. З. Петров. Моделирование физических полей. *Гравитация и теория относительности*, No. 4-5 : 7–21, 1968.
[2] Н. С. Синюков. Геодезические отображения римановых пространств. Москва:Наука, 1979.
[3] І. Курбатова, М.Піструіл. Квазі-геодезичні відображення спеціальних псевдоріманових просторів //Proc. Intern. Geom. Center, 13(3), 18-32, (2020).

Автоморфні функції та алгебри двовимірних сингулярних інтегральних операторів

Мозель В. О.
(вул. Середньофонтанська, 19-Б, кв. 270, 65039, Одеса, Україна)
E-mail: mozel@ukr.net

Нехай D – відкритий одиничний круг комплексної площини. В гільбертовому просторі $L^2(D)$ введемо наступні оператори:

K – добре відомий оператор Бергмана;

$W = W_g$ – унітарний (ізометричний) оператор зваженого зсуву, утворений параболічним або гіперболічним дробно-лінійним перетворенням $g \in G$ круга D в себе, де G – нескінчена циклічна комутативна група, породжена перетворенням g, з однією або двома нерухомими і граничними точками всіх зсувів, що лежать на абсолюті.

Нехай, далі, $\mathfrak{A}$ позначає C^*-алгебру без зсуву, яка породжена операторами, що мають вигляд $A = a(z)I + b(z)K + L$, де I – одиничний, L – компактний, коефіцієнти a, b є автоморфними функціями, тобто задовольняють умовам $a(g(z)) = a(z)$, $b(g(z)) = b(z)$, неперервними на рімановій поверхні групи.

Вивчається C^*-алгебра $\mathfrak{B}$, породжена усіма операторами вигляду

$$B = \sum_{j=-\infty}^{+\infty} A_j W^j$$

де A_j – оператори алгебри $\mathfrak{A}$.

Виявляється, що алгебра $\mathfrak{B}$ є розширенням алгебри $\mathfrak{A}$ за допомогою операторів зсуву W_g, де $g \in G$. Будується алгебра символів та встановлюється критерій фредгольмовості для операторів C^*-алгебри $\mathfrak{B}$.

Канонічні квазі-геодезичні відображення псевдо-ріманових просторів з рекурентно-параболічною структурою

Піструіл М.І.
(ОНУ, Одеса, Україна)
E-mail: margaret.pistruil@gmail.com

Курбатова І.М.
(ОНУ, Одеса, Україна)
E-mail: irina.kurbatova27@gmail.com

В [3] ми досліджували дифеоморфізми псевдо-ріманових просторів, які є квазі-геодезичними відображеннями [1] і водночас майже-геодезичними 2-го типу [2]. Основні рівняння такого відображення (V_n, g_{ij}, F_i^h) і $(\overline{V}_n, \overline{g}_{ij})$ в сумісній за відображенням системі координат (x^i) мають вигляд [3]

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}(x)\delta_{j)}^h + \phi_{(i}(x)F_{j)}^h(x),$$
$$F_{ij} = -F_{ji}, \quad F_{ij} = g_{i\alpha}F_j^\alpha, \quad \overline{F}_{ij} = -\overline{F}_{ji}, \quad \overline{F}_{ij} = \overline{g}_{i\alpha}F_j^\alpha,$$
$$F_\alpha^h F_i^\alpha = 0$$
$$F_{(i,j)}^h = F_{(i}^h q_{j)},$$

де $\overline{\Gamma}_{ij}^h, \Gamma_{ij}^h$ - компоненти об'єктів зв'язності $\overline{V}_n$ і V_n, ψ_i, φ_i - деякі ковектори; "," - знак коваріантної похідної в V_n.

Афінорну структуру, для якої диференціальні рівняння набувають вигляду $F_{(i,j)}^h = F_{(i}^h q_{j)}$, ми називаємо узагальнено-рекурентною, а при $F_{i,j}^h = F_i^h q_j$ - рекурентно-параболічною.

У випадку, коли в основних рівняннях квазі-геодезичного відображення $\psi_i(x) = 0$, його називають канонічним.

Отримана лінійна форма основних рівнянь канонічних квазі-геодезичних відображень рекурентно-параболічних просторів. З її допомогою доведені основні теореми теорії канонічних квазі-геодезичних відображень рекурентно-параболічних просторів, які дають змогу для будь-якого псевдо-ріманового простору (V_n, g_{ij}, F_i^h) з рекурентно-параболічною афінорною структурою однозначно відповісти на питання, допускає він розглядуване відображення чи ні.

Далі розглянуто канонічне квазі-геодезичне відображення рекурентно-параболічного простору (V_n, g_{ij}, F_i^h) на полусиметричний простір $\overline{V}_n$, отже тезор Рімана $\overline{V}_n$ задовольняє умовам

$$\overline{R}_{ijk|[lm]}^h = 0,$$

де "|" - знак коваріантної похідної в $\overline{V}_n$.

Доведена

Теорема 1. *Якщо рекурентно-параболічний простір (V_n, g_{ij}, F_i^h) допускає нетривіальне канонічне квазі-геодезичне відображення на полусиметричний $\overline{V}_n$, то виконується принаймні одна з умов: $\varphi_{i,j} = aF_{ij} - \varphi_i q_j$ або $R_{i\alpha}F_j^\alpha = bF_{ij}$, при деяких інваіантах a, b.*

Література

[1] А. З. Петров. Моделирование физических полей. *Гравитация и теория относительности*, No. 4-5 : 7–21, 1968.
[2] Н. С. Синюков. Геодезические отображения римановых пространств. Москва:Наука, 1979.
[3] І. Курбатова, М.Піструіл. Квазі-геодезичні відображення спеціальних псевдоріманових просторів //Proc. Intern. Geom. Center, 13(3), 18-32, (2020).

Геометрія наближення для простору афінної зв'язності

Покась Сергій Михайлович
(Одеський національний університет імені І. І. Мечникова, Одеса, Україна)
E-mail: pokas@onu.edu.ua

Ніколайчук Анна Олександрівна
(Одеський національний університет імені І. І. Мечникова, Одеса, Україна)
E-mail: nickolaychuck@stud.onu.edu.ua

Розглянемо простір афінної зв'язності без скруту A_n, віднесений до довільної системи координат $\{x^1, x^2, \dots, x^n\}$, з об'єктом зв'язності $\Gamma^h_{ij}(x)$; $M_0(x^h_0)$ — фіксована точка цього простору. Побудуємо новий простір $\tilde{A}_n$, віднесений до координат $\{y^1, y^2, \dots, y^n\}$, зі своїм об'єктом зв'язності $\tilde{\Gamma}^h_{ij}(y)$, який задається співвідношенням

$$\tilde{\Gamma}^h_{ij}(y) = -\frac{1}{3}\underset{0}{R}^h_{.(ij)l}\, y^l, \text{ де } \underset{0}{R}^h_{.ijl} = R^h_{.ijl}(M_0). \tag{1}$$

Якщо система координат у вихідному просторі A_n є канонічною з початком у точці M_0, то об'єкт зв'язності $\tilde{\Gamma}^h_{ij}$ реалізує наближення першого порядку для Γ^h_{ij} вихідного простору і тому відображає геометричні властивості A_n з деяким ступенем точності [1, 4].

Вивчаються деякі властивості простору $\tilde{A}_n$. Зокрема, доведено, що система координат $\{y^1, y^2, \dots, y^n\}$ є рімановою з початком у точці M_0.

Надалі розглядаються аналітичні інфінітезимальні рухи в просторі $\tilde{A}_n$

$$y'^h = y^h + \tilde{\xi}^h(y)\,\delta t, \text{ де } \tilde{\xi}^h(y) \text{— вектор зміщення.} \tag{2}$$

Компоненти вектора $\tilde{\xi}^h(y)$ шукаються у вигляді степеневих рядів.

$$\tilde{\xi}^h(y) \equiv a^h + \sum_{k=1}^{\infty} \underset{k}{a}^h = a^h + \sum_{k=1}^{\infty} a^h_{l_1 l_2 \dots l_k} y^{l_1} y^{l_2} \dots y^{l_k}, \text{ де } a^h, a^h_{l_1 l_2 \dots l_k} \text{— константи.} \tag{3}$$

При дослідженні основних рівнянь [2, 3]

$$L_{\tilde{\xi}}\tilde{\Gamma}^h_{ij}(y) \equiv \frac{\partial^2 \tilde{\xi}^h}{\partial y^i \partial y^j} + \tilde{\xi}^\alpha \frac{\partial \tilde{\Gamma}^h_{ij}}{\partial y^\alpha} + \frac{\partial \tilde{\xi}^\alpha}{\partial y^i}\tilde{\Gamma}^h_{\alpha j} + \frac{\partial \tilde{\xi}^\alpha}{\partial y^j}\tilde{\Gamma}^h_{\alpha i} - \frac{\partial \tilde{\xi}^h}{\partial y^\alpha}\tilde{\Gamma}^\alpha_{ij} = 0 \tag{4}$$

у явному вигляді знайдено вектор $\tilde{\xi}^h(y)$:

$$\tilde{\xi}^h(y) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k!(2k-1)} a^\alpha t^{(k)h}_\alpha, \text{ де} \tag{5}$$

$$t^i_j = \frac{1}{3}\underset{0}{R}^i_{.l_1 l_2 j} y^{l_1} y^{l_2}, \quad t^{(p)i}_j = t^{(p-1)i}_\alpha t^\alpha_j \quad (p = 2, 3, \dots). \tag{6}$$

Доведена абсолютна та рівномірна збіжність цих рядів у деякій області. Вивчається питання про порядок групи Лі розглянутих рухів.

Література

[1] А. П. Норден. Пространства аффинной связности. *М.: Наука*, 1976. – с. 431
[2] И. П. Егоров. Геометрия: Спец. курс для студентов физ.-мат. фак. пед. ин-тов. *М.: Просвещение*, 1979. – с. 256
[3] Движения в пространствах аффинной связности. *ИКУ*, 1965. – 206 с.
[4] А. З. Петров. Новые методы в общей теории относительности. *М.: Наука*, 1966. – с. 496

Про 3F-планарні відображення псевдо-ріманових просторів

Андрій Соловйов
(ОНУ, Одеса, Україна)
E-mail: andrey-solovyov@stud.onu.edu.ua

Ірина Курбатова
(ОНУ, Одеса, Україна)
E-mail: irina.kurbatova27@gmail.com

Юлія Хабарова
(ОНУ, Одеса, Україна)
E-mail: yulia-habarova@stud.onu.edu.ua

Досліджуючи майже контактні многовиди, К.Яно, С.Хоу і В.Чен [1] дійшли до поняття *квадриструктури*, структурний афінор якої задовольняє рівнянню $\phi^4 \pm \phi^2 = 0$.

Ми вивчаємо 3F-планарні відображення [2] псевдо-ріманових просторів (V_n, g_{ij}, F_i^h) і $(\overline{V}_n, \overline{g}_{ij}, \overline{F}_i^h)$ з афінорною структурою певного виду, основні ріняння яких в загальній за відображенням системі координат (x^i) мають вигляд:

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \sum_{s=0}^{3} \overset{s}{q}_{(i}(x) \overset{s}{F}{}_{j)}^h(x),$$

де

$$\overset{\circ}{F}{}_i^h = \delta_i^h, \quad \overset{1}{F}{}_i^h = F_i^h, \quad \overset{2}{F}{}_i^h = \overset{1}{F}{}_i^\alpha \overset{1}{F}{}_\alpha^h, \quad \overset{3}{F}{}_i^h = \overset{2}{F}{}_i^\alpha \overset{1}{F}{}_\alpha^h, \quad \overset{s}{F}{}_i^h(x) = \overset{s}{\overline{F}}{}_i^h(x),$$
$$F_\alpha^h F_\beta^\alpha F_\delta^\beta F_i^\delta + F_\alpha^h F_i^\alpha = 0, \quad g_{i\alpha} F_j^\alpha = -g_{j\alpha} F_i^\alpha, \quad F_{i,j}^h = F_{i|j}^h = 0,$$

Γ_{ij}^h, $\overline{\Gamma}_{ij}^h$ - компоненти об'єктів зв'язності V_n і $\overline{V}_n$, відповідно; $\overset{s}{q}_i(x)$ - деякі ковектори; F_i^h - афінор; $<,>$, $<|>$ - знаки коваріантної похідної в V_n і $\overline{V}_n$.

Ми довели, що за таких умов на афінор простори V_n і $\overline{V}_n$ є локально зведеними і мають вигляд добутку

$$V_n = V_m \times V_{n-m}, \quad \overline{V}_n = \overline{V}_m \times \overline{V}_{n-m},$$

до того ж на компонентах цього добутку 3F-планарне відображення $f : V_n \longrightarrow \overline{V}_n$ індукує F-планарне відображення [3] $f_1 : V_m \longrightarrow \overline{V}_m$ параболічно келерових просторів [3] і F-планарне відображення $f_2 : V_{n-m} \longrightarrow \overline{V}_{n-m}$ еліптично келерових просторів [3].

Література

[1] Yano Kentaro, Houh Chorng-Shi, Chen Bang-Yen. Structures difined by a tensor field ϕ of type (1,1), satisfying $\phi^4 \pm \phi^2 = 0$. *Tensor*, 23(1) : 81–87, 1972.

[2] Р. Дж. Кадем *2F-планарные отображения аффинносвязных и римановых пространств.*,- Дисс. на соиск. учен. степ. к. ф.-м. н. Одес. ОГУ, 1992 108 с.

[3] J. Mikes, A. Vanzurova, I. Hinterleitner *Geodesic Mappings and Some Generalizations.*,- Palacky University, Olomouc, Faculty of Science. Olomouc, 2009.

Дослідження властивостей неперервних обмежених розв'язків систем нелінійних різницево-функціональних рівнянь у гіперболічному випадку

Т. О. Єрьоміна
(Національний технічний кніверситет України «Київський політехнічний інститут ім. І. Сікорського», Київ, Україна)
E-mail: ierominat@ukr.net

О. А. Поварова
(Національний технічний кніверситет України «Київський політехнічний інститут ім. І. Сікорського», Київ, Україна)
E-mail: olena_sivak@ukr.net

Розглядається система нелінійних різницево-функціональних рівнянь вигляду

$$x\left(qt\right)=\Lambda x\left(t\right)+f\left(t,x\left(t+1\right)\right), \tag{1}$$

у випадку, коли виконуються наступні умови:

(1) Λ - дійсна $(n\times n)$-матриця вигляду $\Lambda=diag\left(\Lambda_1,\Lambda_2\right)$, де Λ_1,Λ_2 - дійсні $(p\times p)$ та $(r\times r)$-матриці $(p+r=n)$, $\det\Lambda\neq 0$. $f:\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$,
$f\left(t,x\left(t+1\right)\right)=\left(f^1\left(t,x^1\left(t+1\right),x^2\left(t+1\right)\right),f^2\left(t,x^1\left(t+1\right),x^2\left(t+1\right)\right)\right)$, q - деяка дійсна додатна стала.

(2)
$$\begin{aligned}\left|f^1\left(t,\bar{x}^1,\bar{x}^2\right)-f^1\left(t,\bar{\bar{x}}^1,\bar{\bar{x}}^2\right)\right|\le l_1\left(\left|\bar{x}^1-\bar{\bar{x}}^1\right|+\left|\bar{x}^2-\bar{\bar{x}}^2\right|\right),\\ \left|f^2\left(t,\bar{x}^1,\bar{x}^2\right)-f^2\left(t,\bar{\bar{x}}^1,\bar{\bar{x}}^2\right)\right|\le l_2\left(\left|\bar{x}^1-\bar{\bar{x}}^1\right|+\left|\bar{x}^2-\bar{\bar{x}}^2\right|\right),\end{aligned}$$

де l_1,l_2 - деякі додатні сталі, що залежать від $l(l_1=l_1\left(l\right),l_2=l_2\left(l\right),l_1\to 0,l_2\to 0$ при $l\to 0)$. Тоді система рівнянь (1) запишеться у вигляді

$$\left\{\begin{array}{l} x^1\left(qt\right)=\Lambda_1x^1\left(t\right)+f^1\left(t,x^1\left(t+1\right),x^2\left(t+1\right)\right),\\ x^2\left(qt\right)=\Lambda_2x^2\left(t\right)+f^2\left(t,x^1\left(t+1\right),x^2\left(t+1\right)\right),\end{array}\right. \tag{2}$$

де $x^1=\left(x_1,...,x_p\right),\ x^2=\left(x_{p+1},...,x_{p+r}\right),\ f^1=\left(f_1,...,f_p\right),\ f^2=\left(f_{p+1},...,f_{p+r}\right).$

Богдан Фещенко, [09.05.2022 15:04] Виконавши в (2) взаємно-однозначну заміну змінних

$$\begin{aligned}x_1\left(t\right)=y_1\left(t\right)+\tilde{\gamma}_1\left(t\right),\\ x_2\left(t\right)=y_2\left(t\right)+\tilde{\gamma}_2\left(t\right),\end{aligned}$$

де $\gamma\left(t\right)=\left(\tilde{\gamma}_1\left(t\right),\tilde{\gamma}_2\left(t\right)\right)$ - неперервний обмежений розв'язок системи (2), отримаємо систему рівнянь

$$\left\{\begin{array}{l} y^1\left(qt\right)=\Lambda_1y^1\left(t\right)+F^1\left(t,y^1\left(t+1\right),y^2\left(t+1\right)\right),\\ y^2\left(qt\right)=\Lambda_2y^2\left(t\right)+F^2\left(t,y^1\left(t+1\right),y^2\left(t+1\right)\right).\end{array}\right. \tag{3}$$

Вектор-функції $F^1\left(t,y^1,y^2\right),F^2\left(t,y^1,y^2\right)$ задовольняють умові 2. і $F^1\left(t,0,0\right)\equiv 0,F^2\left(t,0,0\right)\equiv 0$. Для системи (3) доведена наступна теорема.

`Теорема.` *Нехай виконуються умови 1-2 і умови:*

3. $0<\lambda_i<1<\lambda_j,i=1,2,\dots p,j=p+1,2,\dots n,0\le p\le n,q>1;$

4. $\theta=\max\left\{\frac{2l_1}{1-\lambda^*},\frac{2l_2}{\lambda_*-1}\right\}<1,$ *де* $1>\lambda^*>\max\left\{\lambda_i,i=1,\dots,p\right\},1<\lambda_*<\min\left\{\lambda_i,i=p+1,\dots,n\right\}.$

Тоді система рівнянь (3) має сім'ю неперервних обмежених при $t\ge T>0$ *(*T *- деяка достатньо* *велика додатна стала) розв'язків у вигляді рядів*

$$y^1(t) = \sum_{i=0}^{\infty} y_i^1(t), y^2(t) = \sum_{i=0}^{\infty} y_i^2(t),$$

де $y_i^1(t)$, $y_i^2(t)$, $i = 0, 1, ...$ - *деякі неперервні обмежені при* $t \geq T > 0$ *вектор-функції.*

Література

[1] Пелюх Г.П. К теории систем линейных разностных уравнений с непрерывным аргументом // Докл. АН. – 2006. – 407, №5 – С. 600 – 603.

[2] Єрьоміна Т.О. Про побудову неперервних розв'язків систем нелінійних різницево-функціональних рівнянь. // Вісник Київського національного університету імені Т. Шевченка. Серія: Фізико-математичні науки. – 2015. – №2. – С. 71-74.

F-планарні відображення майже симплектичних многовидів

Юлія Хабарова
(ОНУ, Одеса, Україна)
E-mail: yulia-habarova@stud.onu.edu.ua

Ірина Курбатова
(ОНУ, Одеса, Україна)
E-mail: irina.kurbatova27@gmail.com

Андрій Соловйов
(ОНУ, Одеса, Україна)
E-mail: andrey-solovyov@stud.onu.edu.ua

Ми вивчаємо F-планарні відображення (псевдо-)ріманових просторів з афінорною структурою певного типу ([2]).

В рімановому просторі (V_n, g_{ij}) афінор F^h_j визначає симплектичну структуру([3]), якщо поле тензора типу $(0,2)$ $\quad F_{ij} = F^\alpha_j g_{\alpha i}$ задовольняє умовам:

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0, \quad F_{ij} + F_{ji} = 0, \quad F_{ij} = F^\alpha_j g_{\alpha i}, \quad |F^h_i| \neq 0,$$

де знак коваріантної похідної в просторі S_n.

Мы обираємо структуру більш загального типу, відмовляючись від вимоги невирожденості афінора. Будемо називати її *майже симплектичною*, а (псевдо-)ріманoвий простір з такою структурою - *майже симплектичним.*

Далі ми досліджуємо F-планарні відображення псевдо-ріманових просторів V_n і $\overline{V}_n$ в припущенні, что афінор F визначає майже симплектичну структуру на V_n і $\overline{V}_n$. Їх основні рівняння мають вигляд

$$\overline{\Gamma}^h_{ij}(x) = \Gamma^h_{ij}(x) + \psi_{(i}(x)\delta^h_{j)}(x) + \varphi_{(i}(x)F^h_{j)}(x),$$

де $\overline{\Gamma}^h_{ij}, \Gamma^h_{ij}$ - компоненти об'єктів зв'язності $\overline{V}_n$ і V_n, ψ_i, φ_i - деякі ковектори. Доведена

Теорема 1. *Майже симплектичний простір (V_n, g_{ij}) допускає нетривіальне F-планарне відображення тоді і тільки тоді, коли в ньому існує неособливий симетричний тензор a_{ij} типу (0,2), який задовольняє диференціальним рівнянням*

$$a_{ij,k} = -\overset{1}{\varphi}_\alpha F^\alpha_i g_{jk} - \overset{1}{\varphi}_\alpha F^\alpha_j g_{ik} - \overset{1}{\varphi}_i F_{jk} - \overset{1}{\varphi}_j F_{ik},$$

$$F^\alpha_i a_{\alpha j} = -F^\alpha_j a_{\alpha i}$$

при деякому векторі $\varphi^1{}_i \neq 0$.

Далі за допомогою a_{ij} ми отримуємо *інваріантне перетворення*([4]) , яке пару майже симплектичних просторів, що знаходяться в нетривіальному F-планарному відображенні, перетворює в нову пару майже симплектичних просторів, що також знаходяться в нетривиальному F-планарному відображенні, але відповідаючому іншому афінору:

$$\Gamma\big(g, \overline{g}, \varphi, F\big): \quad \big(S_n \overset{\varphi,F}{\longmapsto} \overline{S}_n\big) \quad \longmapsto \quad \big(\overset{1}{S}_n \overset{\varphi^1,F^1}{\longmapsto} \overset{1}{\overline{S}}_n\big).$$

Завдяки цьому з'явилася можливість отримання великої кількості прикладів пар майже симплектичних просторів, які знаходяться в F-планарному відображенні.

Література

[1] J.Mikes, A.Vanzurova, I.Hinterleitner. Geodesic Mappings and Some Generalizations//Palacky University, Olomouc, Faculty of Science. Olomouc, 2009.

[2] Микеш Й., Синюков Н.С. О квазипланарных отображениях пространств аффинной связности // Известия ВУЗов, Математика, (1983), №1(248),56-61.

[3] Фоменко А.Т. Дифференцальная геометрия и топология.Дополнительные главы// М.: из-во Моск. ун-та, (1983).

[4] Синюков Н.С. Геодезические отображениях римановых пространств // Москва "Наука (1979).

Зміст